Soft Risk Disclosure with Feedback Effect

by

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Business Administration

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S. “Vish” Viswanathan

Dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Business Administration in the Graduate School of Duke University

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ABSTRACT

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Abstract

This paper studies firms' optimal *qualitative* disclosure about *hard-to-quantify* risk exposure to affect investors' information acquisition under the feedback effect channel. Based on a model with unknown payoff distribution, disclosure softness, and ambiguity aversion, I find that firms with lower risk exposure disclose more precisely. Particularly, low (medium) exposure firms provide perfect (partially informative) risk disclosures, whereas high exposure firms always disclose vaguely. In addition, the softness of risk disclosure enables firms to induce *different* risk perceptions among informed and uninformed investors with one disclosure, which gives firms the flexibility to separately influence the beliefs of the two groups of investors. Finally, I find that lower cost of information acquisition may improve economic efficiency at the expense of risk disclosure quality.

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Chapter 1

Introduction

While managers have long been regarded as more knowledgeable than the market about their own firms, an emerging literature on informational feedback provides evidence that managers resort to the financial market for information they do not possess.[1] One type of such information is about common risks, such as macroeconomic risks or environmental risks, which firms in an economy or an industry generally need to manage, but have limited knowledge about.

Like any other decision-relevant information compounded in prices, information about common risks comes from the effort of investors to acquire information. However, given the decentralized and costly nature of information acquisition, investors would only choose to learn about the risks *they perceive* as most relevant for their investments. Consequently, it is possible that the risks that are highly relevant for firm value are seldom investigated, while the risks investors do investigate are of minor influence. This would be an inefficient use of resources, not only for individual investors, but also for firms that could have made better decisions by learning from the market.

This paper studies the optimal disclosure about common risk exposure, if firms want to learn from the market about the common risk. Assuming that firms know better than the market about the value-relevance of risks, reflected by their exposure to the risks, firms may have the incentive to affect investors' information acquisition

[1] Prior papers in the area include Luo (2005), Chen et al. (2006), Bakke and Whited (2010), Bai et al. (2016), Zuo (2016), Edmans et al. (2017), Jayaraman and Wu (2018) and Yan (2019), among others.

by strategically disclosing their risk exposure. In addition, disclosure about such risk exposure is assumed to be soft information that is hard to verify.

Based on a model with unknown payoff distribution, soft disclosure, and ambiguity averse investors[2], I find that to encourage information acquisition about common risks, firms with lower exposure to the risks provide more precise risk disclosures. In particular, low (medium) exposure firms provide perfect (partially informative) risk disclosures, whereas high exposure firms always provide uninformative risk disclosures.

In the model, the firm's payoff has two components: the interim performance in the first phase ($v_1 = \beta z + \epsilon$), and the outcome of a risk management action a^* in the second phase ($v_2 = (z - a^*)\sqrt{\beta}$), which equals to the firm's updated expectation of the common risk z based on the interim performance v_1 and the stock price. Because the action is purely risk-hedging and generates zero expected value, investors' uncertainty about firm value stems only from the uncertainty about the common risk.

Assuming that investors do *not* know the realization nor the distribution of the firm's risk exposure β, uncertainty of firm value *perceived* by investors depends on the *perceived* exposure $\hat{\beta}$.[3] By shaping investors' *perceived* exposure[4] with risk disclosure, the firm can therefore affect investors' *perceived* uncertainty, which determines their learning and trading decisions. More importantly, given that the risk disclosure is soft, which is modeled as an *interval* of potential exposure (i.e., $\beta \in [\beta_L, \beta_H]$), two *different* perceptions may be induced among informed and uninformed investors, i.e., $\hat{\beta}^{Informed} \neq \hat{\beta}^{Uninformed}$. In this case, firms have the flexibility to *separately* influence informed and uninformed investors' decisions.

[2] Refer to Lin (2019) for more details and explanations about the framework.

[3] Throughout the paper, an variable with hat $\hat{x}$ represents the perception of the variable x.

[4] In the paper, I use "perceived risk exposure" and "risk perception" interchangeably.

I find that informed and uninformed investors' utilities are both *U-shape* functions of their respective risk perceptions due to two opposite effects of perceptions on *perceived* posterior uncertainty. On one hand, higher risk perception implies that the firm is believed to bear more units of common risk. This is the *risk bearing effect* that increases investors' *perceived* uncertainty. On the other hand, higher risk perception also implies that the firm's interim performance is believed to be a more precise signal of the common risk. As a result, the firm is believed to learn better about the risk from its own performance and hence make better risk management decisions later. This is the *learning effect* of risk perception that decreases investors' *perceived* uncertainty. When risk perception is high, the *learning effect* dominates. Assuming that informed investors only consider the effects of perceptions on posterior uncertainty, both informed and uninformed investors would set their perceptions as close as possible to their respective troughs, which are the levels of perceptions that generate the highest possible perceived uncertainty.

The firm cares about both the expectation and the variance of the terminal value, and pursues higher price informativeness by shaping investors' perceptions with risk disclosure. Investors' perceptions affect price informativeness by changing both the *ex-ante* incentive to acquire private information and the *ex-post* incentive to trade on private information. However, the second effect on *ex-post* trading is already considered when investors decide whether to learn in the first place, so focus on the first effect on the incentive to acquire information. Intuitively, investors have more incentive to acquire information when the comparative advantage of getting informed is higher. The *perceived* comparative advantage is higher when the perceived uncertainty of getting informed is low, and/or the perceived uncertainty of staying uninformed is high.

In other words, to induce larger *perceived* comparative advantage, the firm prefers to be perceived as *more* risky by the uninformed investors, but *less* risky by the informed investors. Note that given no disclosure, both informed and uninformed investors tend to set their perceptions at the level that generates the highest possible perceived riskiness. This implies that given no disclosure, uninformed investors form risk perceptions in a way that the firm likes, whereas informed investors form perceptions in a way that the firm dislikes. Therefore, if the firm has the flexibility to exert separate influences on informed and uninformed investors, it will disclose perfectly to informed investors to curb their unfavorable perceptions, but disclose vaguely to uninformed investors to indulge their favorable perceptions.

The magnitude of flexibility to exert separate influences on informed and uninformed investors is determined by the firm's true exposure. When exposure is moderate lying between the two troughs of informed and uninformed investors, the firm has most flexibility since informed and uninformed investors choose different bounds of the disclosure as perceptions. Particularly, knowing that the informed investors would choose the upper bound, the firm sets the upper bound at the true exposure, rendering informed investors' perception at the truth. In contrast, knowing that the uninformed investors would choose the lower bound, the firm sets the lower bound beneath the trough of uninformed investors, rendering uninformed investors' perception set at their own trough. Therefore, for firms with medium exposure, it is optimal to provide *partially informative* risk disclosure, which enables some but not all investors to form correct risk perceptions.

However, for low or high exposure that lies below or above both troughs of investors, the firm is less able to exert separate influences on informed and uninformed investors, because both groups tend to choose the same bound of disclosure as perceptions. When the two perceptions are constrained to be the same, i.e., $\hat{\beta}^{Informed} = \hat{\beta}^{Uninformed} = \hat{\beta}$, the firm always prefers a lower $\hat{\beta}$. In this case, the *risk* *bearing effect* of perception $\hat{\beta}$ to increase the units of risk has the same impact on the informed and uninformed investors. The only difference between getting informed and staying uninformed is the perceived uncertainty of each unit of risk.

To strengthen the perceived comparative advantage of the private information to resolve uncertainty of each unit of risk, it is optimal for the firm to make the public disclosure of interim performance appear less informative, which can be achieved by lower perception $\hat{\beta}$ and hence weaker *learning effect*. Moreover, investors tend to set perceptions at the lower bound of disclosure when firms have high exposure, but set perceptions at the upper bound when firms have low exposure. Combining investors' perception choices and firms' incentive to induce lower perceptions, it is therefore optimal for firms with low (high) exposure to decrease the upper (lower) bound of disclosures, implying perfect (uninformative) disclosure under low (high) exposure.

While above analyses are based on the assumption that informed investors only consider the effects on posterior uncertainty when choosing risk perceptions, I obtain similar results after relaxing the assumption to incorporate the effect of informed investors' perception on posterior expectation. Further analyses demonstrate the effects of information acquisition cost. On one hand, lower information acquisition cost always enables firms to promote higher price informativeness, which improves economic efficiency by helping firms to make better risk management decisions. In other words, lower information cost improves real efficiency by improving revelatory price efficiency, which is defined as the ability of price to "reveal information necessary for decision makers to take value-maximizing actions" in Bond et al. (2012). On the other hand, lower acquisition cost may impair the quality of risk disclosure, making it harder for investors to form correct risk perceptions. In conclusion, there is a potential trade-off between information efficiency and economic efficiency.

This paper has two main contributions. First, the paper contributes to the risk disclosure literature as it sheds light on the cross-sectional variation of firms' qualitative (i.e., soft) risk disclosures. As discussed in Lin (2019), the type of disclosures, quantitative or qualitative, is an important determinant of firms' incentives to provide risk disclosures. While prior theories apply to quantitative risk disclosures about quantifiable risk exposure (Jorgensen and Kirschenheiter, 2003; Heinle and Smith, 2017; Heinle et al., 2018; Smith, 2019), this paper adds to the literature by modeling qualitative risk disclosures. Different from Lin (2019), which reveals the time-series variation of soft risk disclosure under different macroeconomic conditions, this paper finds that firms with different levels of exposure to common risks have different incentives to provide soft risk disclosures.

Second, the paper adds to the feedback effect literature by studying how firms can use soft risk disclosures to guide market learning. In a setting in which firms learn from stock prices about the common risks, I identify the conditions under which firms provide precise disclosures about risk exposure to encourage information acquisition. This is new to the literature as prior studies generally show that disclosures suppress information acquisition and therefore impair the feedback effect (Gao and Liang, 2013; Jayaraman and Wu, 2018). One closely related paper is Smith (2019), which also studies the impact of risk disclosures on feedback effect. However, we explore different types of risk disclosures, and should be better understood as complements rather than conflicting theories. This also reiterates the need to cater theoretical models to depict different types of risk reporting. More detailed discussion is in the next section.

Some policy implications can be drawn. First, requiring more precise risk disclosure may harm real efficiency, if the risks are unfamiliar to everyone, and about which firms can learn from the financial markets. Second, firms with high exposure generally provide less precise risk disclosures to encourage market learning. To the extent that the purpose to mandate risk disclosures is to reveal to investors the most exposed entities in the economy, this suggests that the objective of mandatory risk disclosure may not always be achieved, at least in the realm of *qualitative* risk disclosures. Finally, the possibility for firms to use the same financial report to induce *different* risk perceptions among investors in an *anticipated* way may be of special concern for regulators, given the existing effort such as the Regulation Fair Disclosure (Reg FD) to preclude selective disclosures.

The paper is organized as follows. In Chapter 2, I discuss the related literature. In Chapter 3, I establish the model and analyze the main results. In Chapter 4, I include policy implications and potential empirical applications. Chapter 5 is the conclusion.

Chapter 2

Related Literature

The paper is related to the literature on informational feedback. The idea that firms learn from financial markets to make better decisions can date back as early as Hayek (1945). For a long time, researchers propose that because market prices have the ability to aggregate all sorts of information from investors, they may contain additional information that firm managers need but do not have perfect knowledge themselves, generating the feedback effect of market prices (Dye and Sridhar 2002; Bond et al. 2012).

Some recent studies explore the impact of mandatory disclosure on feedback effect. Gao and Liang (2013) show theoretically that mandatory disclosure may increase stock liquidity at the expense of feedback effect, because investors facing lower information asymmetry have less incentive to acquire information in the first place. Jayaraman and Wu (2018) provides empirical support for the theory, showing that investment-q sensitivity decreases after segment reporting becomes mandatory in US, with the effect concentrated in firms with more informed trading.

While prior research focuses on the impact on feedback effect of mandatory disclosure about *firm cash flows*, this paper studies mandatory disclosure about *firm riskiness*, and finds that risk disclosure could *enhance* or *impair* feedback effect under different conditions. Closely related, Smith (2019) also studies the impact of risk disclosure on information acquisition. However, this paper significantly differs from his in many aspects.

First, this paper focuses on *soft* (i.e., qualitative) risk disclosure, which is modeled as an interval of potential risk exposure, instead of one signal of risk as in Smith (2019), which is more applicable to *hard* (i.e., quantitative) risk disclosure. In practice, many mandatory risk disclosures, such as risk factor disclosure, cybersecurity risk disclosure, and management discussion (MD&A) of risks, are qualitative and narrative. Furthermore, the model shows that the softness of risk disclosure enables firms to strategically foster *different* risk perceptions among equally ambiguity/risk averse investors. In other words, the softness makes firms' risk reporting much more flexible than the truth-telling-or-nothing decision as in Smith (2019), despite the mandatory nature of such risk disclosure.

Second, this paper studies disclosure about exposure to systematic risks, whereas Smith (2019) focuses on disclosure about idiosyncratic risks. One special feature about systematic risk is that, the more exposed is a firm to the risk, the more precisely its performance reflects the risk. To the extreme, firms with high exposure to a systematic risk (i.e., $performance = \beta\ systematic\ risk + idiosyncratic\ risk$, with $\beta \to +\infty$) can perfectly learn the systematic component from their own performance. In the model, this is called the *learning effect* of perceived risk exposure. The *learning effect* of risk exposure makes the firm's demand for information *non-monotonic* in risk exposure, and is very important in the analysis under information acquisition.

Due to the differences in setting, this paper has some different results compared with Smith (2019). For instance, Smith (2019) finds that information acquisition (and hence price informativeness) is an *inverse U-shape* function of disclosed risk. In this paper, things get more complicated since the softness of risk disclosure enables firms to induce *different* risk perceptions among informed and uninformed investors, which is to some extent equivalent to disclosing two levels of risk among informed and uninformed investors. Therefore, given the endogenous determination of information acquisition, price informativeness is a *U-shape* function of informed investors' risk perception, but an *inverse U-shape* function of uninformed investors' perception. Furthermore, even given the relations between price informativeness and risk perceptions, firms' optimal risk disclosure is not straightforward because how the two levels of risk perceptions are induced by the same disclosure differs across scenarios. To summarize, this paper and Smith (2019) explore different types of risk disclosures, and should be better understood as complements rather than conflicting theories.

This paper is also related to the large stream of literature about the relation between public and private information, which has shown the conditions for informational substitution[1], or informational complementarity[2]. In particular, this paper identifies the conditions under which precise disclosure about risk exposure can encourage or suppress more information acquisition about the risk.

[1] Diamond (1985), Fischer and Stocken (2010), Amador and Weill (2010), Han and Yang (2013), Chen et al. (2014).

[2] Kim and Verrecchia (1994a), Demski and Feltham (1994), McNichols and Trueman (1994), Boot and Thakor (2001), Hellwig and Veldkamp (2009), Goldstein and Yang (2015), Xue and Zheng (2018).

Chapter 3

The Model

There is one firm and a continuum of investors in the economy. Investors can invest in one risk-free asset with zero return and one risky asset (one firm). Investors are denoted by subscript j. The firm and all investors are risk averse and have negative exponential utility. Risk aversion of the firm is γ_f and risk aversion of all investors is γ. At the beginning of period, the firm carries out a project with the exposure to risk factor z as $\beta > 0$.[1] While the firm knows its own exposure, investors do not observe β and do not even know its distribution.

The assumption that investors do *not* know the distribution of risk exposure is meant to reflect investors' limited knowledge about firms' risk exposure. It is not rare that investors may not be aware of the existence or materiality of certain risk exposure before firms disclose it. Moreover, Kravet and Muslu (2013) find that stock volatility and trading volume increase around and after textual risk disclosures, suggesting that risk disclosures reveal *unknown* risk factors. In a word, investors have much less knowledge about firms' risk exposure compared with other firm features such as revenue or capital structure. The assumption of *unknown* distribution of risk exposure to investors reflects this fact.

The project's terminal cash flow v has two components: the interim performance and the risk management outcome. The interim performance, $v_1 = \beta z + \epsilon$, is determined by the firm's exposure to the common risk factor z and the idiosyncratic risk ϵ[2] . z and ϵ are independent and follow normal distribution with variances σ_z^2 and

[1] Results can be easily generalized to the case of negative exposure.

[2] Since there is only one firm in this model, it may be difficult to separate common and idiosyn-

σ_ϵ^2. Throughout the paper, I denote the variance of a random variable x by σ_x^2 and the precision by θ_x. z has mean $\overline{z}$ and ϵ has mean zero .

Furthermore, the risk management outcome depends on an action a, which is taken after the firm updates its belief about z at the second phase. It is assumed that the action $a \leq E(z|\textit{Firm's Information})$, ensuring that the firm takes the action to hedge the risk rather than to speculate. This action could be interpreted as the firm's effort to mitigate its exposure to the common risk. The firm's terminal value v is the sum of the interim performance and later risk management outcome:

$$v = v_1 + v_2 = \beta z + \epsilon - (z - a)\sqrt{\beta} \tag{3.1}$$

In the interim period, the firm offers two types of disclosures. First, the firm is required to disclose perfectly the interim performance v_1, which can be interpreted as mandatory disclosure of earnings. In addition to the earnings disclosure, the firm has to disclose its exposure to the common risk (i.e., β), which is mandatory risk disclosure.

To capture the soft nature of many risk disclosures, I model the disclosure of β as an interval of $[\beta_L, \beta_H]$ covering the true β. This should be understood as risk disclosures allow or are compatible with any risk perceptions by investors within the interval, but not necessarily that firms disclose their risk exposure in the form of intervals. The smaller is the range, the more precise.[3] In practice, exposure to many

cratic risks. Another way to interpret this may be risks that the firm is aware or unaware of its own exposure.

[3] Even if risk disclosure is mandatory, firms still have large discretion in deciding the disclosure precision, given the difficulty for regulators to judge whether the disclosure is informative enough. According to a report by the Government Accountability Office (GAO, 2018), SEC "faces constraints in reviewing climate-related and other (risk) disclosures because it primarily relies on information that companies provide". SEC reviewers can "request additional information or clarification from companies ..., but a company may claim that the risk-related issue raised by SEC is not material and hence does not need further disclosure".

systematic risks such as cybersecurity risks, environmental risks, or uncertainty of regulatory changes, is usually difficult to convey in one or several metrics, making disclosures about such risk exposure predominantly qualitative. Qualitative risk disclosures are by nature soft. Compared with quantitative disclosures with metrics or numerical analysis, qualitative risk disclosures leave more space for multiple interpretations by investors.

After the firm releases its financial report containing $\{v_1, [\beta_L, \beta_H]\}$, investors decide whether to pay a cost of τ to receive a signal about the common risk $s_z = z + \varphi$. The noise term φ follows normal distribution with zero mean and variance of σ_φ^2. Then conditional on respective information sets, informed and uninformed investors decide their risk perception $\hat{\beta}_j$, and the demand for the firm's stock x_j, to maximize expected utility. Here, investors are assumed to be ambiguity averse and follow the max-min optimization rule. First, given any demand for stock x, investors choose a perceived level of the firm's risk exposure, $\hat{\beta}(x) \in [\beta_L, \beta_H]$, that renders the lowest expected utility. Then they compare all the worst-scenario utilities under different pairs of x and $\hat{\beta}(x)$, and choose the optimal demand for stock x that maximizes the worst-scenario utility. Figure 3.1 shows the timeline of the model.

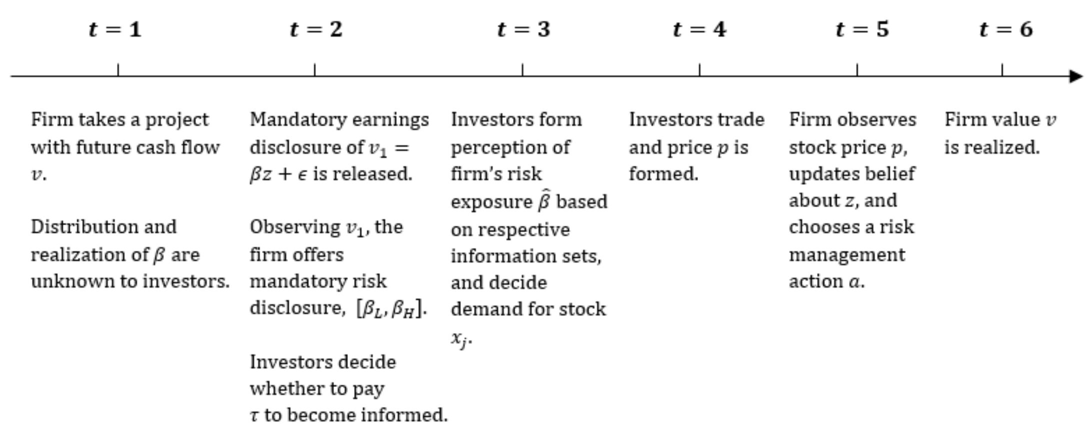

Figure 3.1: Timeline

The model is solved by backward induction. I first look at the firm' optimal risk management action a based on its updated belief of the risk z at $t = 5$. Conditional on investors being informed or uninformed, I solve their choices of perceived risk exposure $\hat{\beta}_j$ and stock demand x_j at $t = 3$, along with the equilibrium stock price at $t = 4$. Then, I look at investors' decision whether to get informed at $t = 2$. Finally, anticipating investors' responses, I solve the firm's optimal risk disclosure at $t = 2$.

3.1 Benchmark: No Information Acquisition

I first look at a benchmark case in which *no* investor acquires information about the common risk at $t = 2$. I will resume to the case with information acquisition by investors in the next section to see how firms' disclosure decisions would be changed by their incentive to learn from the market about the common risk.

3.1.1 Firm's Risk Management Action and Optimal Disclosure Policies

Start with the firm's risk management decision at $t = 5$. Given the firm's disclosure policy and investors' responses, the firm's optimal action is $a^* = E(z|\textit{Firm's Information})$. Because the stock price is uninformative of z, the only signal about z is the interim performance $\frac{v_1}{\beta} = z + \frac{\epsilon}{\beta}$, with precision $\beta^2\theta_\epsilon$. Define the firm's information set at time t as I_t^f. At $t = 5$, $I_5^f = \{v_1, \beta\}$. The firm's optimal action is the weighted average of signals as:

$$a^* = E(z|I_5^f) = \frac{\theta_z}{\theta_z + \beta^2\theta_\epsilon}\overline{z} + \frac{\beta^2\theta_\epsilon}{\theta_z + \beta^2\theta_\epsilon}\frac{v_1}{\beta}$$

Assume that the firm cares about the mean and variance of its terminal value v[4], and therefore chooses the risk disclosure policy at $t = 2$ that maximizes:

$$
\begin{aligned}
&\max_{\{\beta_L,\beta_H\}} E(v|I_2^f) - \frac{\gamma_f Var(v|I_2^f)}{2} \\
&=E\left\{v_1 - \sqrt{\beta}[z - E(z|I_5^f)]|I_2^f\right\} - \frac{\gamma_f \beta Var[z - E(z|I_5^f)|I_2^f]}{2} \\
&=v_1 - \frac{\gamma_f \beta Var(z|I_5^f)}{2} \qquad (3.2)
\end{aligned}
$$

where $I_2^f = \{v_1, \beta\}$.

Because there is no information acquisition by investors, the firm's information set I_5^f is *not* affected by any disclosures. Therefore, the firm's utility is only affected by its true risk exposure β. As a result, the firm is *indifferent* about any risk disclosures, and would perfectly disclose its true exposure β.[5]

Proposition 1. *Under no information acquisition by investors, the firm is indifferent about any disclosures, and would therefore provide perfect risk disclosure.*

Although the firm's optimal risk disclosure is independent of investors' responses under no information acquisition, it is still useful to go though the analysis of investors' decisions at $t = 3$ to lay out the effects of investors' risk perceptions in the benchmark case. This facilitates later comparison of the effects of investors' risk perceptions with and without information acquisition.

4 This differs from Lin (2019) in which the firm cares about the mean and variance of the stock price.

5 If the firm cares about stock price, it would also be optimal to give perfect risk disclosure under no private information acquisition by investors, for reasons other than indifference. Refer to the appendix for an analysis when the firm cares about stock price.

3.1.2 Investors

Given the firm's risk management strategy at $t = 5$, investors decide their risk perception $\hat{\beta}_j$ and stock demand x_j at $t = 3$. Conditional on a choice of perceived risk exposure $\hat{\beta}_j$, define investors' information set as $I_j = \{v_1, \hat{\beta}_j\}$. Ambiguity averse investors solve the following max-min optimization problem (subscript j for risk perception is omitted):

$$\begin{aligned} &\max_{x_j} \quad \min_{\hat{\beta}} \quad E(v - p|I_j)x_j - \frac{\gamma Var(v|I_j)}{2} x_j^2 \\ &\text{s.t.} \quad E(v|I_j) = v_1, \; Var(v|I_j) = \frac{\hat{\beta}}{\theta_z + \hat{\beta}^2 \theta_\epsilon} \end{aligned} \tag{3.3}$$

Since the expected firm value is always the disclosed earnings v_1, risk perception $\hat{\beta}$ affects investors' utility only through its influence on the posterior uncertainty $Var(v|I_j)$. In particular, $Var(v|I_j)$ is an *inverse U-shape* function of risk perception $\hat{\beta}$ due to two opposite effects of $\hat{\beta}$: on one hand, the more exposed is a firm perceived, the firm is believed to bear more units of common risk z. This is the *risk-bearing effect* of risk perception, which increases investors' uncertainty $Var(v|I_j)$. On the other hand, the more exposed is a firm perceived, the more precise is the interim performance as a signal of the risk for the firm, because the perceived noise $\frac{\sigma_\epsilon^2}{\hat{\beta}^2}$ decreases with $\hat{\beta}$. This enables the firm to take better risk management action later, which ultimately decreases investors' posterior variance. This is the *learning effect* of risk perception, which decreases investors' uncertainty $Var(v|I_j)$.

When risk perception is high enough, the *learning effect* dominates the *risk-bearing effect*. The opposite is true when perception is low. Consequently, $Var(v|I_j)$ is an *inverse U-shape* function of risk perception $\hat{\beta}$, rendering investors' utility a *U-shape* function of $\hat{\beta}$, with its trough at $\hat{\beta} = \frac{\sigma_\epsilon}{\sigma_z}$. Due to ambiguity aversion, investors would set their perceptions at the level within any disclosed interval $[\beta_L, \beta_H]$, that is *closest* to the trough and hence generates the *highest* posterior variance. The result is formally stated in Proposition 2.

Proposition 2. *Ambiguity averse investors' perceived risk exposure is:*

$$\hat{\beta}^* = \begin{cases} \beta_L, & \beta_L \geq \frac{\sigma_\epsilon}{\sigma_z} \\ \frac{\sigma_\epsilon}{\sigma_z}, & \frac{\sigma_\epsilon}{\sigma_z} \in (\beta_L, \beta_H) \\ \beta_H, & \beta_H \leq \frac{\sigma_\epsilon}{\sigma_z} \end{cases} \tag{3.4}$$

and their stock demand is:

$$x_j^* = \frac{E(v-p|I_j)}{\gamma Var(v|I_j)} = \frac{(v_1 - p)(\theta_z + \hat{\beta^*}^2 \theta_\epsilon)}{\gamma \hat{\beta}^*} \tag{3.5}$$

No inertia zone exists where $x_j^* = 0$ *under any public signals.*

All proofs can be found in the appendix. One special feature of ambiguity-averse models is that an area of *inertia* may exist in which $x_j^* = 0$ for any signals in certain parameter space. However, Proposition 2 proves the existence of an equilibrium with no inertia area, and characterizes the optimal risk perception and stock demand of investors.

Figure 3.2 demonstrates how the firm's risk disclosure $[\beta_L, \beta_H]$ can affect investors' risk perception $\hat{\beta}$. Define understatement (overstatement) as a disclosed interval with its midpoint below (above) the true exposure. Suppose the firm's true exposure is 7, and the trough is $\hat{\beta} = \frac{\sigma_\epsilon}{\sigma_z} = 5$. Compare two disclosures with the same range (i.e., same precision), either an understatement of $\beta \in [4, 8]$, or an overstatement of $\beta \in [6, 10]$. Both disclosures are truth telling as they cover the true exposure $\beta = 7$.

Given investors' strategy to set risk perception *closest* to the trough, the upper

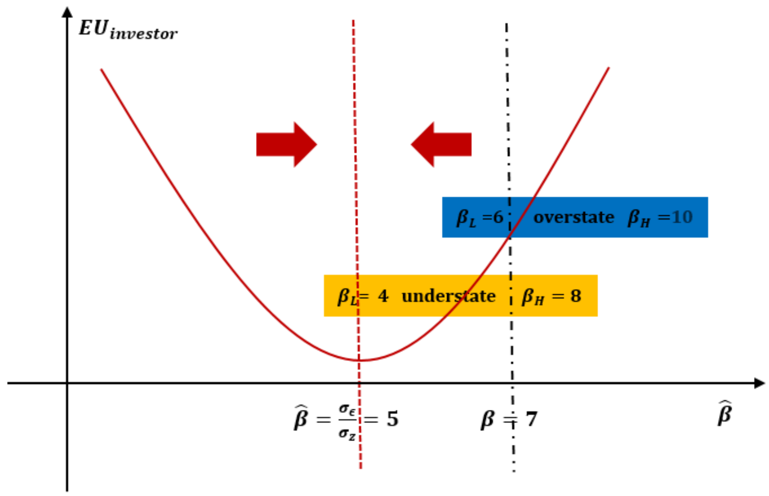

Figure 3.2: Investors' Risk Perceptions

bound of disclosure does *not* matter in this case. Risk perception would be 5 given a disclosure of $\beta \in [4, 8]$, and 6 given a disclosure of $\beta \in [6, 10]$. Depending on its preference of investors' risk perception $\hat{\beta}$, the firm could release risk disclosures to induce a preferred perception. For instance, if the firm prefers to be perceived at lower exposure, it would choose $\beta \in [4, 8]$ over $\beta \in [6, 10]$. However, note that the lowest possible perception it can induce is at the trough 5, instead of its lower bound 4.

3.1.3 Stock Price

Assume the supply of stock is 1 and the demand of liquidity trader is q, with mean zero and variance of σ_q^2. Based on the market clearing condition and the optimal demand of investors, the equilibrium stock price is:

$$p = v_1 + \frac{\gamma\hat{\beta}}{\theta_z + \hat{\beta}^2\theta_\epsilon}(q - 1) \tag{3.6}$$

which is consistent with the classic result as the sum of the expected firm value and risk premium.

To summarize, in the benchmark case under no information acquisition by investors, risk averse firms are *indifferent* about any disclosures, and therefore would disclose perfectly. This resonates with prior literature that generally predicts perfect risk disclosure (Jorgensen and Kirschenheiter, 2003; Heinle and Smith, 2017; Heinle et al., 2018).[6] However, empirical evidence finds that firms' risk disclosures can be generic and uninformative (e.g, Kravet and Muslu, 2013; Bao and Datta, 2014; Dyer et al., 2017), suggesting that firms may *lack* the incentive to provide perfect qualitative risk disclosure under certain conditions.

In the next section, I introduce information acquisition by investors to the benchmark case, and study how this new element may change the firm's risk disclosure policy.

3.2 Model with Information Acquisition

When investors are allowed to acquire information about the common risk, the stock price becomes informative for the firm to learn about the risk and take wiser risk management action later. In this case, risk disclosure affects not only stock price, but also real firm value through the feedback channel, i.e., how effectively firms can learn from market prices about common risk factors.

Everything is the same as the benchmark case, except that now investors are given the option at $t = 2$ to pay a cost of τ to receive a signal about the common risk $s_z = z + \varphi$. The noise term φ follows normal distribution with mean zero and variance of σ_φ^2. If investors decide not to get informed, they observe no additional signal but can still learn from the stock price. The proportion of informed traders in the market is denoted by Ω. Henceforth, the firm is denoted by script f, informed

[6] The only exception is Heinle et al. (2018) showing that firms may not disclose risk exposure if exposure uncertainty induces positive skewness.

investors by script I and uninformed investors by U. As before, I solve the model by backward induction.

3.2.1 Firm's Risk Management Action

Start from the firm's risk management decision at $t = 5$, fixing the firm's disclosure policy, investors' responses and the pricing function. Since the stock price is informative of the common risk z, the firm's action is now based on the price, too. Define the price signal about z as $\tilde{p}$, and price informativeness as θ_p. The endogenous determination of θ_p will be discussed in detail later. It follows that the firm's information set is $I_5^f = \{v_1, \tilde{p}, \beta\}$.

The firm's optimal action is a weighted average of the three available signals:

$$a^* = E(z|I_5^f) = \frac{\theta_z}{\theta_z + \beta^2\theta_\epsilon + \theta_p}\overline{z} + \frac{\beta^2\theta_\epsilon}{\theta_z + \beta^2\theta_\epsilon + \theta_p}\frac{v_1}{\beta} + \frac{\theta_p}{\theta_z + \beta^2\theta_\epsilon + \theta_p}\tilde{p}$$

3.2.2 Informed and Uninformed Investors

Next are investors' choices of their risk perceptions and stock demands at $t = 3$, fixing the firm's risk disclosure and the proportion of informed investors Ω. As in the benchmark case, investors face a max-min problem as:

$$\max_{x_j^k} \min_{\hat{\beta}^k} \; E(v - p|I^k)x_j^k - \frac{\gamma Var(v|I^k)}{2}(x_j^k)^2, \;\; k \in \{I, U\}$$

with conditional mean and variance of firm value v depending on the respective information sets of informed and uninformed investors. The information set of the informed investors is $I^I = \{v_1, p, s_z\}$, and the uninformed is $I^U = \{v_1, p\}$. Therefore the worst scenarios for informed and uninformed investors generally differ. This implies that the same risk disclosure $[\beta_L, \beta_H]$ could induce different risk perceptions among informed and uninformed investors, i.e., $\hat{\beta^I} \neq \hat{\beta^U}$.

Specifically, for the uninformed investors, their expected value of $E(v|I^U)$ is fixed at v_1 and not affected by risk perception $\hat{\beta^U}$, because they have the same information set as the firm (i.e., $I_5^f = I^U$). Therefore, uniformed investors' utility is affected by $\hat{\beta^U}$ only through the variance term, as in the benchmark case:

$$E(v|I^U) = v_1 + \sqrt{\hat{\beta^U}} E[E(z|v_1, p) - z|v_1, p] = v_1, \;\; Var(v|I^U) = \frac{\hat{\beta^U}}{\theta_z + \hat{\beta^U}^2 \theta_\epsilon + \theta_p} \tag{3.7}$$

Again, due to the *risk-bearing effect* and the *learning effect* of risk perception, the posterior variance $Var(v|I^U)$ is an *inverse U-shape* function of $\hat{\beta^U}$. Therefore, uninformed investors' utility is a *U-shape* function of $\hat{\beta^U}$, with trough at $T_\beta^U = \sqrt{\frac{\theta_z + \theta_p}{\theta_\epsilon}}$. Uninformed investors would set perception $\hat{\beta^U}$ at the point *closest* to the trough, within any disclosure $[\beta_L, \beta_H]$.

However, for the informed investors, not only the variance $Var(v|I^I)$, but also the expected value $E(v|I^I)$ is affected by perception $\hat{\beta^I}$, because now they have more precise information than the firm (i.e., $I_5^f \subset I^I$). Therefore, informed investors' utility is affected by $\hat{\beta^I}$ through both the level effect and the variance effect:

$$E(v|I^I) = v_1 + \sqrt{\hat{\beta^I}} E[E(z|v_1, p) - z|v_1, p, s_z] \neq v_1 \tag{3.8}$$

$$Var(v|I^I) = \frac{\hat{\beta^I}}{\theta_z + \hat{\beta^I}^2 \theta_\epsilon + \theta_\varphi} \tag{3.9}$$

Now that $E(v|I^I)$ is affected by $\hat{\beta^I}$, it is possible that informed investors may not buy (sell) when price is not low (high) enough, which is the classic inertia equilibrium in ambiguity aversion models. To see the main driving forces, I first assume that when informed investors choose perception $\hat{\beta}^I$, they only consider its effects on the posterior variance term $Var(v|I^I)$, neglecting the level effect on $E(v|I^I)$. I will relax this assumption in the next section to incorporate the level effect. Again $Var(v|I^I)$ is an *inverse U-shape* function of $\hat{\beta}^I$, and hence informed investors' utility is a *U-shape* function of $\hat{\beta^U}$, with trough at $T^I_\beta = \sqrt{\frac{\theta_z+\theta_\varphi}{\theta_\epsilon}}$. Therefore, informed investors would set perception $\hat{\beta}^I$ at the point *closest* to the trough, within any disclosure $[\beta_L, \beta_H]$.

Proposition 3 formally states the choices of risk perceptions and stock demands by informed and uninformed investors.

Proposition 3. *Assume that informed investors only consider the posterior variance when they choose perception $\hat{\beta}^I$ (with no assumption for the uninformed investors):*
1) The utilities of informed and uninformed investors are both U-shape functions of respective risk perceptions;
2) There exist thresholds $T^k_\beta, k \in \{I, U\}$, such that the optimal risk perceptions of investors $\hat{\beta}^k, k \in \{I, U\}$ are:

$$\hat{\beta}^k = \begin{cases} \beta_L, & \beta_L \geq T^k_\beta \\ T^k_\beta, & T^k_\beta \in (\beta_L, \beta_H) \\ \beta_H, & \beta_H \leq T^k_\beta \end{cases} \tag{3.10}$$

in which $T^U_\beta = \sqrt{\frac{\theta_z+\theta_p}{\theta_\epsilon}}$, and $T^I_\beta = \sqrt{\frac{\theta_z+\theta_\varphi}{\theta_\epsilon}}$;
3) The optimal stock demand is:

$$x^k_j = \frac{E(v-p|I^k)}{\gamma Var(v|I^k)} \tag{3.11}$$

Note that while both informed and uninformed investors have *U-shape* preferences of respective risk perceptions, the uninformed investors have a *lower* trough than the informed, i.e., $T_\beta^U \leq T_\beta^I$. This is because uniformed investors have no private information, and rely more on the firm's earning disclosure $v_1 = \beta z + \epsilon$ to learn about z. Therefore compared with informed investors, uninformed investors are harmed more (in terms of higher posterior variance) by a lower perception, which implies that the public disclosure of v_1 is believed to be less informative about z.

3.2.3 Price Informativeness

Conjecture that the stock price takes the following linear form:

$$p = w_0 + w_1 \overline{z} + w_2 v_1 + w_3 s_z + w_4 q \tag{3.12}$$

Rewrite the price p to $\tilde{p}$[7] as a signal of z, with mean of $\bar{z}$, and variance σ_p^2:

$$\tilde{p} \equiv \frac{p - (w_0 + w_1 \overline{z} + w_2 v_1)}{w_3} = z + \varphi + \frac{w_4}{w_3} q \tag{3.13}$$

$$\sigma_p^2 \equiv \sigma_\varphi^2 + \left(\frac{w_4}{w_3}\right)^2 \sigma_q^2 \tag{3.14}$$

Given the expressions of investors' stock demands in Proposition 5, the market clearing condition is:

$$\Omega \frac{E(v|I^I) - p}{\gamma Var(v|I^I)} + (1 - \Omega) \frac{E(v|I^U) - p}{\gamma Var(v|I^U)} + q = 1 \tag{3.15}$$

Based on the market clearing condition, the coefficients w_i ($i \in \{0, ..., 4\}$) in the pricing function (3.12) could be identified by solving a fixed-point problem. The key element to identify is $iratio \equiv |\frac{w_4}{w_3}|$, which determines the informativeness of price

[7] Variable with tilde, $\tilde{x}$, is used to denote the transformed version of the variable x.

about z, i.e., $\theta_p = (\sigma_\varphi^2 + iratio^2\sigma_q^2)^{-1}$:

$$iratio \equiv \left|\frac{w_4}{w_3}\right| = \frac{\gamma\sqrt{\hat{\beta}^I}}{\Omega\theta_\varphi}; \quad \theta_p = \frac{\theta_\varphi}{1 + \frac{\gamma^2\hat{\beta}^I}{\Omega^2\theta_\varphi}\sigma_q^2} \tag{3.16}$$

Intuitively, price informativeness always increases with the proportion of informed investors Ω and the precision of their private signal θ_φ, and decreases with investors' risk aversion γ. Conditional on Ω, higher perception of informed investors $\hat{\beta}^I$ leads to lower price informativeness θ_p, because risk-averse informed investors would have lower demand if the asset is believed to bear more units of risk. This implies informed investors' less aggressive trading on private information, rendering less information incorporated in price. This is the *informed trading effect* of $\hat{\beta}^I$. In addition, the *informed trading effect* does not apply to uninformed investors' perception $\hat{\beta^U}$, because uniformed investors have *no* private information about z, and hence make no contribution to price informativeness conditional on Ω.

Rearranging Equation (3.15) yields the expression of price as:

$$p = \left[\frac{w^I}{w^I + w^U}E(v|I^I) + \frac{w^U}{w^I + w^U}E(v|I^U)\right] + \frac{1}{w^I + w^U}(q-1) \tag{3.17}$$

where $w^I \equiv \frac{\Omega}{\gamma\hat{\beta}^I Var(z|I^I)}$ and $w^U \equiv \frac{1-\Omega}{\gamma\hat{\beta^U} Var(z|I^U)}$. As usual, the term $\frac{q-1}{w^I+w^U}$ represents the risk premium.

3.2.4 Firm's Optimal Risk Disclosure

Now I turn to the firm's risk disclosure choice based on its information set $I_2^f \equiv \{v_1, \beta\}$ at $t = 2$. Given the risk management action as $a^* = E(z|v_1, p, \beta)$, the expected firm value conditional on I_2^f is always v_1. So the firm chooses risk disclosure only to decrease the conditional variance $Var(v|I_2^f)$ by maximizing price informativeness about the common risk θ_p. As Expression (3.16) shows, θ_p is a function of the equilibrium proportion of informed investors Ω^*. Equilibrium Ω^* solves the indifference condition equating the expected *perceived* utility of informed and uninformed investors:

$$E\left[\frac{E(v-p|I^I,\hat{\beta^I})^2}{2\gamma Var(v|I^I,\hat{\beta^I})}\right]e^{-2\gamma\tau} = E\left[\frac{E(v-p|I^U,\hat{\beta^U})^2}{2\gamma Var(v|I^U,\hat{\beta^U})}\right] \tag{3.18}$$

It is important to note that when investors decide whether to pay for the private information at $t=2$, they can *rationally expect* their risk perceptions at $t=3$ as $\hat{\beta^I}$ if getting informed, or as $\hat{\beta^U}$ if staying uninformed. This is because the choice of $\hat{\beta^I}$ and $\hat{\beta^U}$ only depends on the firm's risk disclosure $[\beta_L, \beta_H]$ and the troughs $T^k_\beta, k \in \{I, U\}$ (Proposition 3), both of which are public information after the firm's disclosures at $t=2$. Simplifying the indifference condition yields $e^{-2\gamma\tau}\hat{Var}(v|I^U,\hat{\beta^U}) = \hat{Var}(v|I^I,\hat{\beta^I})$, which is:

$$e^{-2\gamma\tau}\frac{\hat{\beta^U}}{\theta_z + \hat{\beta^U}^2\theta_\epsilon + \theta_p} = \frac{\hat{\beta^I}}{\theta_z + \hat{\beta^I}^2\theta_\epsilon + \theta_\varphi} \tag{3.19}$$

in which τ is the cost to become informed.

Intuitively, the left hand side (LHS) of Equation (3.19) represents the *perceived* benefit of getting informed, whereas the right hand side (RHS) represents the *perceived* cost: the higher is the perceived variance of staying uninformed (i.e., $\hat{Var}(v|I^U,\hat{\beta^U})$ on the LHS), the more benefit to get informed for the sake of avoiding $\hat{Var}(v|I^U,\hat{\beta^U})$. In contrast, the higher is the perceived variance after getting informed (i.e., $\hat{Var}(v|I^I,\hat{\beta^I})$ on the RHS), the more costly (or equivalently the less beneficial) to get informed.

The firm's risk disclosure affects investors' perception $\hat{\beta^k}$, $k \in \{I, U\}$, and hence affects investors' information acquisition by changing both the *perceived* benefit and the *perceived* cost of getting informed, which eventually influences the firm's utility by changing price informativeness about the risk. This is the feedback effect of risk disclosure incremental to the benchmark case. It follows that to encourage more information acquisition, the firm prefers to be perceived as *more* risky by the uninformed investors, but *less* risky by the informed investors. Therefore, it is optimal for the firm to disclose in a way that induces *higher* $\hat{Var}(v|I^U, \hat{\beta}^U)$ for the sake of higher *perceived* benefit of getting informed, but *lower* $\hat{Var}(v|I^I, \hat{\beta}^I)$ for the sake of lower *perceived* cost of getting informed.

To state the point formally, reorganize Equation (3.19) to obtain an expression of the price informativeness θ_p in equilibrium. To maximize firm value, the firm chooses its risk disclosure to maximize the equilibrium θ_p as:

$$\max_{\{\beta_L, \beta_H\}} \ \theta_p = \frac{e^{-2\gamma\tau}\hat{\beta}^U(\theta_z + {\hat{\beta}^I}^2\theta_\epsilon + \theta_\varphi)}{\hat{\beta}^I} - \theta_z - {\hat{\beta}^U}^2\theta_\epsilon \tag{3.20}$$

$$\text{s.t.} \ \ \theta_p \in \left[0, \frac{\theta_\varphi}{1 + \frac{\gamma^2\hat{\beta}^I\sigma_q^2}{\theta_\varphi}}\right] \tag{3.21}$$

The firm's optimal risk disclosure $[\beta_L^*, \beta_H^*]$ determines investors' risk perceptions $\hat{\beta}^I$ and $\hat{\beta}^U$, which in turn determine both θ_p^* and Ω^*. The upper (lower) bound of θ_p is reached when $\Omega = 1$ ($\Omega = 0$) in Expression (3.16).

Before solving the firm's risk disclosure policy, let's first look at the firm's preference of investors' risk perceptions $\hat{\beta}^I$ and $\hat{\beta}^U$, which is equivalently the relation between price informativeness and risk perception.

Lemma 1. *The firm's expected utility, equivalently price informativeness θ_p, is:*
1) U-shape function of informed investors' risk perception $\hat{\beta}^I$ with its trough at $T^I_{\beta f}$, and inverse U-shape function of uninformed investors' risk perception $\hat{\beta}^U$ with its

peak at $T^U_{\beta f}$*:*

$$\frac{\partial\theta_p}{\partial\hat{\beta}^I}\begin{cases}\geq 0, & \hat{\beta}^I \geq T^I_{\beta f}\\ < 0, & \hat{\beta} < T^I_{\beta f}\end{cases}, \quad \frac{\partial\theta_p}{\partial\hat{\beta}^U}\begin{cases}\leq 0, & \hat{\beta}^U \geq T^U_{\beta f}\\ > 0, & \hat{\beta}^U < T^U_{\beta f}\end{cases} \tag{3.22}$$

in which $T^I_{\beta f} = \sqrt{\frac{\theta_z+\theta_\varphi}{\theta_\epsilon}}$*, and* $T^U_{\beta f} = \frac{e^{-2\gamma\tau}(\theta_z+\hat{\beta^I}^2\theta_\epsilon+\theta_\varphi)}{2\theta_\epsilon\hat{\beta}^I} = T^U_\beta$*.*

2) When $\hat{\beta}^I$ *and* $\hat{\beta}^U$ *are constrained to be the same at* $\hat{\beta}$*,* θ_p *degenerates to a decreasing function of* $\hat{\beta}$*.*

Lemma 1 shows that the firm has *opposite* preferences of $\hat{\beta}^I$ and $\hat{\beta}^U$, mainly due to the endogenous determination of the proportion of informed investors Ω^*. Specifically, risk perceptions $\hat{\beta}^k, k \in \{I, U\}$ can affect price informativeness θ_p by changing: 1) the *ex-ante* incentive for investors to get informed, and hence the equilibrium portion of informed investors Ω^*; and/or, 2) the *ex-post* willingness to trade, conditional on Ω^*. However, the second channel about the trading incentive is already embedded in the first channel about the learning incentive, because when investors decide whether to acquire information, they have compared the expected trading profits of being informed or uninformed. In conclusion, risk perceptions $\hat{\beta}^k, k \in \{I, U\}$ affect price informativeness mainly by changing investors' *ex-ante* incentive to learn, which is affected by both the perceived benefit and cost of getting informed in Equation (3.19).

As discussed above, investors have stronger incentive to learn if the perceived benefit of getting informed is higher (i.e., higher $\hat{Var}(v|I^U, \hat{\beta}^U)$), and/or the perceived cost is lower (i.e., lower $\hat{Var}(v|I^I, \hat{\beta}^I)$). Therefore, to encourage information acquisition, the firm prefers to be perceived as more risky by uninformed investors (higher $\hat{Var}(v|I^U, \hat{\beta}^U)$), but be perceived as less risky by informed investors (lower $\hat{Var}(v|I^I, \hat{\beta}^I)$). Combining with the fact that $Var(v|I^U)$ and $Var(v|I^I)$ are *inverse*

U-shape functions of respective perceptions $\hat{\beta^U}$ and $\hat{\beta^I}$ (Proposition 3), the firm has an *inverse U-shape* preference of $\hat{\beta^U}$, but a *U-shape* preference of $\hat{\beta^I}$.

Conditional on $\hat{\beta}^U$, the firm's utility is *lowest* when informed investors' perception is set at $T^I_{\beta f} = \sqrt{\frac{\theta_z+\theta_\varphi}{\theta_\epsilon}}$, which is exactly the point when informed investors have the highest perceived variance. In contrast, conditional on $\hat{\beta}^I$, the firm's utility is *highest* when uninformed investors' perception is set at $T^U_{\beta f} = \frac{e^{-2\gamma\tau}(\theta_z+\hat{\beta^I}^2\theta_\epsilon+\theta_\varphi)}{2\theta_\epsilon\hat{\beta^I}}$, which is the point when uninformed investors have the highest perceived variance. This can be verified by calculating the price informativeness at this point, $\theta^*_p = \frac{e^{-4\gamma\tau}(\theta_z+\hat{\beta^I}^2\theta_\epsilon+\theta_\varphi)^2}{4\theta_\epsilon\hat{\beta^I}^2} - \theta_z$, rendering uninformed investors' trough $T^U_\beta = \sqrt{\frac{\theta_z+\theta_p}{\theta_\epsilon}} = \frac{e^{-2\gamma\tau}(\theta_z+\hat{\beta^I}^2\theta_\epsilon+\theta_\varphi)}{2\theta_\epsilon\hat{\beta^I}}$, which is equal to the firm's trough $T^U_{\beta f}$.

Lemma 1 also shows that when the two perceptions are constrained to be the same $\hat{\beta^I} = \hat{\beta^U} = \hat{\beta}$, the firm prefers lower perception $\hat{\beta}$. The reason is that when $\hat{\beta^I} = \hat{\beta^U} = \hat{\beta}$, the *risk-bearing* effects of risk perception as a multiplier of risk are exactly the same for both groups of investors, rendering the only difference between the informed and uninformed investors being the uncertainty of each unit of risk. Facing each unit of risk, informed investors learn from earnings v_1 and private information s_z, whereas uninformed investors learn from v_1 and price, with earnings as the common knowledge. Therefore, to highlight the perceived informational advantage of informed investors, it is optimal to decrease the perceived informativeness of the public signal v_1. This can be achieved by impairing the *learning* effects of perception $\hat{\beta}$, in other words, lowering $\hat{\beta}$.

Based on the firm's preference of risk perceptions, we can now study its risk disclosure policy to shape investors' perceptions. Note that under the assumption that informed investors' risk perception choice only consider the variance effect neglecting the level effect, the trough/peak of the firm's preferences of risk perceptions $T^k_{\beta f}, k \in \{I, U\}$ are *equal* to the troughs of investors' preferences $T^k_\beta, k \in \{I, U\}$ respectively. Given no disclosure, uninformed and informed investors tend to form perceptions that generates the highest perceived uncertainty $Var(v|I^U)$ and $Var(v|I^I)$. While this is perfectly aligned with the firm's preference to be perceived *more* risky by uninformed investors, it is against the firm's preference to be perceived *less* risky by informed investors.

Therefore, if the firm has the flexibility to exert separate influences on the two groups of investors, it is ideal to disclose perfectly to the informed investors in order to curb their unfavorable perceptions, but to disclose vaguely to the uninformed investors in order to indulge their favorable perceptions. However, the flexibility for the firm to do so depends on the level of its true exposure β, or more precisely, the relative position of β and the two troughs T_β^I and T_β^U. It is also affected by the information acquisition cost τ. Define $\psi(\tau) \equiv \sqrt{\frac{1}{2e^{2\gamma\tau}-1}} < 1$. Proposition 4 summarizes the firm's risk disclosure choice and the resultant risk perceptions of investors.

Proposition 4. *The firm's optimal risk disclosure and the resultant risk perceptions depends on the level of* β*:*

1) Under $\boldsymbol{\beta \geq T_\beta^I}$*: The firm discloses* $\forall \beta_L \leq T_\beta^U = e^{-2\gamma\tau} T_\beta^I$*, and* $\forall \beta_H \geq \beta$*. This induces risk perceptions* $\hat{\beta}^I = T_\beta^I > \hat{\beta}^U = T_\beta^U$*.*

2) Under $\boldsymbol{\beta \in \left(\psi(\tau)T_\beta^I, T_\beta^I\right)}$*: The firm discloses* $\forall \beta_L \leq T_\beta^U = \frac{e^{-2\gamma\tau}(\theta_z + \beta^2\theta_\epsilon + \theta_\varphi)}{2\theta_\epsilon\beta}$*, and* $\beta_H^* = \beta$*, which leads to risk perceptions of* $\hat{\beta}^I = \beta > \hat{\beta}^U = T_\beta^U$[8]*.*

3) Under $\boldsymbol{\beta \leq \psi(\tau)T_\beta^I}$*: The firm discloses* $\forall \beta_L \leq \beta$*, and* $\beta_H^* = \beta$*. This induces*

[8] Here, the firm's optimal upper bound is $\beta_H^* = \beta$. To prevent unraveling, I introduce an exogenous noise to the risk disclosure that is beyond the firm's control, rendering the reported upper bound randomly above β. This noise could stem from the financial reporting system, or from investors' intellectual process to interpret soft information. The same treatment is used in Proposition 7 and Proposition 6 below without further explanation. Another way is to assume an exogenous bound of precision $[\beta_L^c, \beta_H^c]$, which the firm's disclosure can *not* be more precise than. This implies that given the firm's chosen disclosure as $[\beta_L^*, \beta_H^*]$, the disclosure *received* by investors is the union of the firm's choice and the exogenous bound. Refer to Lin (2019) for more details.

risk perceptions $\hat{\beta}^I = \hat{\beta}^U = \beta$.

The firm's optimal risk disclosure policy is shown in Figure 3.3. Here, "perfect" ("vague") disclosure implies that given the disclosure, *all* (*no*) investors form(s) correct risk perceptions, and "partially informative" disclosure implies that given the disclosure, only some investors form correct risk perceptions.

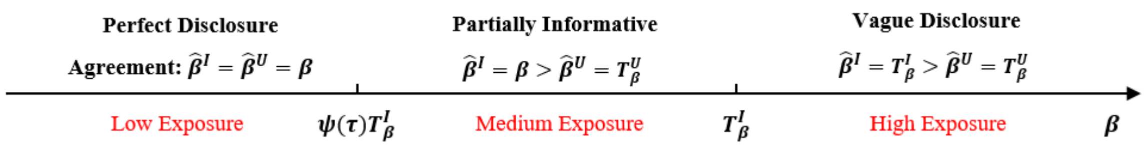

Figure 3.3: Optimal Risk Disclosure

Proposition 4 shows that to encourage market learning, it is optimal to disclose perfectly for firms with low exposure $\beta \leq \psi(\tau)T^I_\beta$, to provide partially informative disclosure for firms with medium exposure $\beta \in (\psi(\tau)T^I_\beta, T^I_\beta)$, and to disclose vaguely for firms with high exposure $\beta \geq T^I_\beta$. $\beta = \psi(\tau)T^I_\beta$ is the indifference point when β would be equal to the resultant trough of uninformed investors T^U_β. Any β above (below) $\psi(\tau)T^I_\beta$ would be higher (lower) than the respective equilibrium T^U_β.

According to Lemma 1, it is ideal for the firm to be perceived by uninformed investors at a level of $\hat{\beta}^U$ *closer* to their own trough T^U_β, but perceived by informed investors at a level of $\hat{\beta}^I$ *further* from their trough T^I_β, if the two perceptions can differ. As discussed above, knowing that investors would set perceptions at their own troughs when there is no disclosure (or equivalently given a disclosure with an infinite interval of $\beta \in (0, +\infty)$), the firm only has the incentive to affect informed investors' risk perceptions, but not that of the uninformed. In other words, if the firm can disclose *separately* to the uninformed and informed investors, it would always disclose vaguely to the uninformed, but disclose perfectly to the informed, because given an interval, the uninformed (informed) investors would always choose the perception that is most (least) favorable for the firm.

However, it is not always possible for the firm to send *separate* signals to the two groups of investors by disclosing the same interval. The flexibility to do so depends on the firm's true exposure β. When β lies between the two troughs of investors, i.e., $\beta \in (T_\beta^U, T_\beta^I)$, the uninformed investors would set perception at the lower bound β_L (until β_L decreases to T_β^U), and the informed investors would set perception at the upper bound β_H (until β_H increases to T_β^I). In this case, the firm obtains most flexibility, and would set an uninformative lower bound $\forall \beta_L \leq T_\beta^U$, but a perfect upper bound $\beta_H^* = \beta$. Therefore, firms with medium exposure would provide partially informative disclosure.

When β is low enough or high enough, the firm is less flexible. In particular, for the least exposed firms with $\beta \leq T_\beta^U$, the lower bound β_L will always be neglected by investors, rendering β_H the only useful tool for firms to shape perception. When β_H is set below T_β^U, both groups of investors form the same perceptions at $\hat{\beta}^I = \hat{\beta}^U = \beta_H$. In this case, it is optimal to set $\beta_H^* = \beta$, because the firm prefers lower perception when the two perceptions are constrained to be the same. Of course, the firm could also set β_H above T_β^U, rendering $\hat{\beta}^I = \beta_H$ and $\hat{\beta}^U = T_\beta^U$. However, in this case it is optimal for the firm to set β_H further away from T_β^I, i.e., $\beta_H^* = T_\beta^U$, rendering the two perceptions the same again. Therefore, the second approach is worse than perfect disclosure. In conclusion, it is optimal for the least exposed firms to disclose perfectly.

In contrast, for the most exposed firms with $\beta \geq T_\beta^I$, the upper bound β_H will be neglected by investors, rendering β_L the only useful tool for firms to shape perception. Following the same rationale, when β_L is set above T_β^I, both groups of investors form the same perceptions at $\hat{\beta}^I = \hat{\beta}^U = \beta_L$. In this case, it is optimal to set $\beta_L^* = T_\beta^I$, because the firm prefers lower perception when the two perceptions are constrained to be the same. The firm could also set β_L below T_β^I, rendering $\hat{\beta}^I = T_\beta^I$ and $\hat{\beta}^U = \beta_L$. More precisely, the firm would set $\forall \beta_L \leq T_\beta^U$, which brings improvement to the first option of $\beta_L^* = T_\beta^I$. In conclusion, it is optimal for the most exposed firms to disclose vaguely with $\forall \beta_L \leq T_\beta^U$.

Given the firm's optimal risk disclosure, investors' risk perception can be shown as a function of true exposure (Figure 3.4). To encourage market learning, firms provide useful information about their risk exposure so that investors can *partially* distinguish firms with different β. In particular, informed investors are able to form correct perceptions for firms with low and medium exposure $\beta \leq T_\beta^I$. Compared with informed investors, uninformed investors' risk perceptions are less precise: first, they have a smaller range of correct perception, i.e., $[0, (\psi(\tau)T_\beta^I]$, with $\psi(\tau) < 1$; second, their perception $\hat{\beta}^U$ is even *negatively* related to β, when firms have medium exposure $\beta \in (\psi(\tau)T_\beta^I, T_\beta^I)$. The result is consistent with the idea that the firm has less incentive to correct uninformed investors' risk perceptions because their perceptions generally go wrong in the direction that favors the firm. In contrast, the firm has more incentive to correct informed investors' perceptions because their perceptions would go wrong in the direction that harms the firm.

The result also shows the benefits related to information acquisition. First, increasing the precision of private information θ_φ would expand the correct perception ranges for both informed and uninformed investors, since T_β^I increases with θ_φ. When private information gets extremely precise, i.e., $\theta_\varphi \to +\infty$, the firm would always provide at least partially informative risk disclosure in order to encourage market learning about the risk. This complementary result of disclosure and information acquisition is new to the feedback effect literature, which generally finds that firms would provide less disclosure to elicit information acquisition. Second, lowering the information acquisition cost τ would be beneficial for uninformed investors. As $\tau \to 0$, $\psi(\tau) \to 1$,

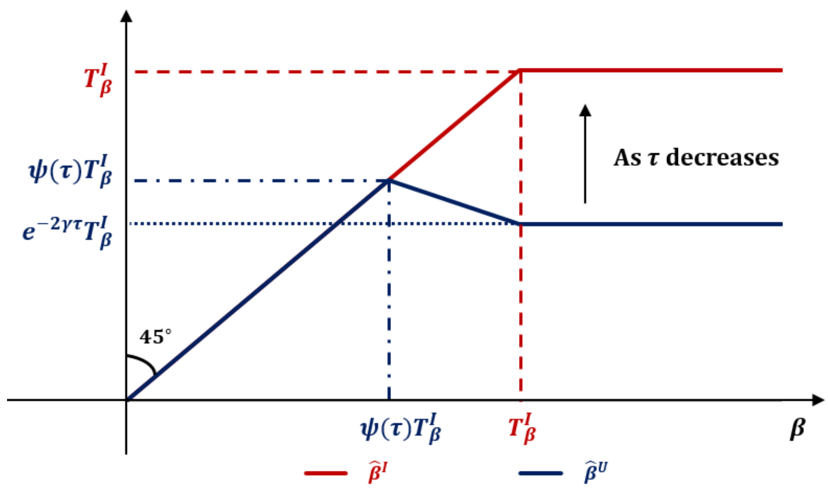

Figure 3.4: Risk Perceptions of Informed and Uninformed Investors

expanding uninformed investors' correct range towards $[0, T_\beta^I)$, and hence dwindling the range of *inverse* perception with $\hat{\beta}^U$ being *negatively* related to β. Finally, the results show a complementary relation between public and private information, as more effective acquisition of private information contributes to an expanded zone of perfect risk disclosure.

Last but not the least, investors disagree when firms offer imprecise risk disclosures. Specifically, the divergence of risk perception $|\hat{\beta}^I - \hat{\beta}^U|$ strictly increases with real exposure in the medium interval $\beta \in (\psi(\tau)T_\beta^I, T_\beta^I)$. Fixing the precision of private information θ_φ, the divergence increases with the cost of information acquisition τ. The disagreement result resonates with the prior literature that disclosures could increase rather than decrease disagreement, if disclosures are *differentially costly* for investors to interpret (Indjejikian, 1991; Kim and Verrecchia, 1994b). In this paper, however, disagreement (in the form of risk perception divergence) is caused by firms' strategic risk disclosures and investors' different information endowments, fixing the cost and ability to interpret soft risk disclosures.

3.3 Relaxed assumption on informed investors' choice of risk perception

Analyses above are based on the assumption that informed investors only consider the posterior variance when they choose risk perception $\hat{\beta}^I$, neglecting the level effect of $\hat{\beta}^I$, represented by the term $\sqrt{\hat{\beta}^I}E[E(z|v_1,p)-z|v_1,p,s_z]$ in Equation (3.8). In this section, I allow informed investors to incorporate the level effect of $\hat{\beta}^I$ in the choice of risk perception, and find generally similar results as in the restricted case[9].

When the level effect of perception on $\sqrt{\hat{\beta}^I}E[E(z|v_1,p)-z|v_1,p,s_z]$ is incorporated, things get much more complicated, because the term is the difference of two weighted averages of signals with each weight being a non-linear function of $\hat{\beta}^I$. To keep the model tractable, I assume that informed investors' choice to buy or sell does *not* depend on risk perceptions: when informed investors choose perceptions, they act *as if* they have already decided to buy or sell depending on the sign of $v_1 - p$.

Conditional on buying ($v_1 - p \geq 0$) or selling ($v_1 - p < 0$), the level effect of perception $\hat{\beta}^I$ matters only to the extent that how much it increases the magnitude of the order to buy (sell) when $v_1 - p$ is positive (negative): when investors buy, they tend to choose lower $\sqrt{\hat{\beta}^I}$ to achieve the *least positive* level effect (i.e., $\sqrt{\hat{\beta}^I}\ *$ *non-negative signal*) to limit the magnitude of buying, and hence limit the potential *perceived* profit of buying; Similarly, when investors sell, they choose lower $\sqrt{\hat{\beta}^I}$ to achieve the *least negative* level effect (i.e., $\sqrt{\hat{\beta}^I}\ *$ *non-positive signal*) to limit the magnitude of selling, and hence limit the potential *perceived* profit of sales.

Therefore, under such assumption that informed investors' choice to buy or sell does not depend on risk perceptions, there is no inertia equilibrium[10]. What is more,

9 Hereafter, I use "the restricted case" to refer to the case assuming that informed investors only consider the posterior variance when they choose risk perception $\hat{\beta}^I$.

10 Without this assumption, there will be inertia equilibrium in which investors may not trade

considering the level effect of perceptions, informed investors now have an extra tendency to set a *lower* perception $\hat{\beta}^I$ in order to limit the potential *perceived* benefit from the level effect, in addition to their tendency to maximize the *perceived* posterior variance by setting perception closer to a trough $\sqrt{\frac{\theta_z+\theta_\varphi}{\theta_\epsilon}}$ as in the restricted case. Consequently, informed investors' utility is still a *U-shape* function of perception $\hat{\beta}^I$, with a trough $T^I_\beta < \sqrt{\frac{\theta_z+\theta_\varphi}{\theta_\epsilon}}$.

Proposition 5 summarizes investors' optimal choice of risk perceptions $\hat{\beta}^k, k \in \{I, U\}$, and stock demands x^k_j. The choices of uninformed investors are exactly the same as in Proposition 4, with the only change being the informed investors.

Proposition 5. *Given that the private information is highly precise (i.e., θ_φ is high):*
1) The utilities of informed and uninformed investors are both U-shape functions of respective risk perceptions;
2) There exist thresholds $T^k_\beta, k \in \{I, U\}$, such that the optimal risk perceptions of investors $\hat{\beta}^k, k \in \{I, U\}$ are:

$$\hat{\beta}^k = \begin{cases} \beta_L, & \beta_L \geq T^k_\beta \\ T^k_\beta, & T^k_\beta \in (\beta_L, \beta_H) \\ \beta_H, & \beta_H \leq T^k_\beta \end{cases} \tag{3.23}$$

in which $T^U_\beta = \sqrt{\frac{\theta_z+\theta_p}{\theta_\epsilon}}$, and $T^I_\beta < \sqrt{\frac{\theta_z+\theta_\varphi}{\theta_\epsilon}}$;
3) The optimal stock demand is:

$$x^k_j = \frac{E(v-p|I^k)}{\gamma Var(v|I^k)} \tag{3.24}$$

within certain level of private information, which makes the price piecewise linear in the private signal of s_z, with a constant price in certain interval of s_z. However, such equilibrium are very difficult to identify in this model given that the expectation term $E(v|I^I)$ is *not* linear in $\hat{\beta}^I$. To keep the model tractable, I assume the independence between the buy/sell decision and risk perception.

Importantly, because of informed investors' extra tendency to set a *lower* perception $\hat{\beta}^I$ in order to limit the potential *perceived* benefit from the level effect, their trough T^I_β is lower than $\sqrt{\frac{\theta_z+\theta_\varphi}{\theta_\epsilon}}$, the level of $\hat{\beta}^I$ that would yield the highest posterior variance $\hat{Var}(v|I^I)$. This also implies that the trough of uninformed investors $T^U_\beta = \sqrt{\frac{\theta_z+\theta_p}{\theta_\epsilon}}$ may be above or below T^I_β. The relative positions of these two troughs of informed and uninformed investors matter a lot for the firm's optimal risk disclosure policy.

Given the investors' optimal risk perceptions and stock demands, the price informativeness, investors' choice to become informed, and the firm's respective preferences of risk perceptions $\hat{\beta}^I$ and $\hat{\beta}^U$ are the same as in the restricted case. As Lemma 1 shows, when informed and uninformed investors are not constrained to be the same, the firm's utility is *lowest* when informed investors have the highest perceived variance, but is *highest* when the uninformed investors have the highest perceived variance. When informed and uninformed investors are not constrained to be the same, the firm prefers lower perception.

The main difference from the restricted case lies in the firm's optimal risk disclosure, which depends on the relative positions of the two troughs T^I_β and T^U_β. In equilibrium, the relative positions of T^I_β and T^U_β depend on the cost of information acquisition τ. Specifically, the indifference condition (Equation 3.19) indicates that *ceteris paribus*, price informativeness in equilibrium θ^*_p decreases with τ. This is intuitive in that the higher is the cost to acquire information, the less incentive for investors to learn in the first place, and the lower is θ^*_p. Furthermore, θ^*_p monotonically increases the trough of uninformed investors' utility $T^U_\beta = \sqrt{\frac{\theta_z+\theta^*_p}{\theta_\epsilon}}$. Consequently, there exists a threshold T_τ such that for $\forall\tau > T_\tau$, $\psi(\tau) < \frac{T^I_\beta}{\sqrt{\frac{\theta_z+\theta_\varphi}{\theta_\epsilon}}} < 1$, rendering the equilibrium $T^U_\beta(\tau)$ below the trough of informed investors' utility T^I_β, and vice versa. $\psi(\tau) = \sqrt{\frac{1}{2e^{2\gamma\tau}+1}}$.

The following discussion is separated into two cases when the information cost is high (i.e., $\tau > T_\tau$), or low (i.e., $\tau \leq T_\tau$). Proposition 6 summarizes the firm's risk disclosure choice and the resultant risk perceptions of investors when the cost is high.

Proposition 6. *When the cost of information is high (i.e., $\tau > T_\tau$), the troughs satisfy $T^I_{\beta f} > T^I_\beta > T^U_\beta = T^U_{\beta f}$.*

1) ***Under*** $\boldsymbol{\beta \geq T^I_\beta}$*: The firm discloses* $\forall \beta_L \leq T^U_{\beta f} = \frac{e^{-2\gamma\tau}(\theta_z + T^{I2}_\beta \theta_\epsilon + \theta_\varphi)}{2\theta_\epsilon T^I_\beta}$*, and* $\forall \beta_H \geq \beta$*. This induces risk perceptions* $\hat{\beta}^I = T^I_\beta > \hat{\beta}^U = T^U_{\beta f}$*.*

2) ***Under*** $\boldsymbol{\beta \in \left(\psi(\tau)\sqrt{\frac{\theta_z+\theta_\varphi}{\theta_\epsilon}}, T^I_\beta\right)}$*: The firm discloses* $\forall \beta_L \leq T^U_{\beta f} = \frac{e^{-2\gamma\tau}(\theta_z + \beta^2 \theta_\epsilon + \theta_\varphi)}{2\theta_\epsilon \beta}$*, and* $\beta^*_H = \beta$*. This induces risk perceptions* $\hat{\beta}^I = \beta > \hat{\beta}^U = T^U_{\beta f}$*.*

3) ***Under*** $\boldsymbol{\beta \leq \psi(\tau)\sqrt{\frac{\theta_z+\theta_\varphi}{\theta_\epsilon}}}$*: The firm disclose* $\forall \beta_L \leq \beta$ *and* $\beta^*_H = \beta$*. This induces risk perceptions* $\hat{\beta}^I = \hat{\beta}^U = \beta$*.*

The firm's optimal risk disclosure policy when the information cost is high is shown in Figure 3.5.

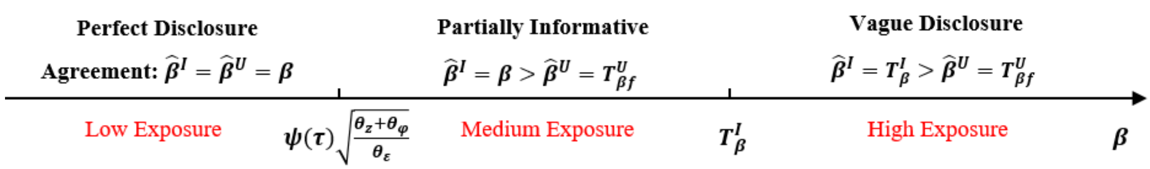

Figure 3.5: Risk Disclosure under High Information Cost ($\tau > T_\tau$)

The results are obtained under high precision of private information θ_φ. When the information acquisition cost is high enough with $\tau > T_\tau$, the troughs satisfy $T^I_\beta > T^U_\beta$, so the firm's optimal risk disclosure and investors' perceptions follow the same pattern as in the restricted case. In addition, lower information acquisition cost τ would always expand the area of perfect risk disclosure in this case, suggesting complementarity between public and private information.

Next, I explore the firm's risk disclosure when the information acquisition cost is low, i.e., $\tau \leq T_\tau$. In this case, the troughs of investors satisfy $T^I_\beta \leq T^U_\beta$, which is new to the restricted case. Proposition 7 gives the firm's optimal risk disclosure, and the resultant risk perceptions of investors when the information acquisition cost is low.

Proposition 7. *When the cost of information is low (i.e., $\tau \leq T_\tau$), the troughs in investors' utilities satisfy $T^I_\beta \leq T^U_\beta$.*

1) Under $\boldsymbol{\beta \geq \frac{e^{-2\gamma\tau}(\theta_z + T_\beta^{I2}\theta_\epsilon + \theta_\varphi)}{2\theta_\epsilon T^I_\beta}}$*: The firm discloses $\forall \beta_L \leq T^I_\beta$, and $\forall \beta_H \geq \beta$. This induces risk perceptions of $\hat{\beta}^I = T^I_\beta < \hat{\beta}^U = T^U_{\beta f} = \frac{e^{-2\gamma\tau}(\theta_z + T_\beta^{I2}\theta_\epsilon + \theta_\varphi)}{2\theta_\epsilon T^I_\beta}$.*

2) Under $\boldsymbol{\beta \in \left(T^I_\beta, \frac{e^{-2\gamma\tau}(\theta_z + T_\beta^{I2}\theta_\epsilon + \theta_\varphi)}{2\theta_\epsilon T^I_\beta}\right)}$*: The firm discloses $\beta^*_L = \beta$, and $\forall \beta_H \geq T^U_{\beta f} = \frac{e^{-2\gamma\tau}(\theta_z + \beta^2\theta_\epsilon + \theta_\varphi)}{2\theta_\epsilon \beta}$. This induces risk perceptions $\hat{\beta}^I = \beta \leq \hat{\beta}^U = T^U_{\beta f} < \frac{e^{-2\gamma\tau}(\theta_z + T_\beta^{I2}\theta_\epsilon + \theta_\varphi)}{2\theta_\epsilon T^I_\beta}$.*

3) Under $\boldsymbol{\beta \leq T^I_\beta}$*, there exist a threshold $T_{\tau 1}$ such that:*

a. If τ is low enough with $\tau < T_{\tau 1}$, the firm discloses $\forall \beta_L \leq \beta$, and $\forall \beta_H \geq T^U_{\beta f} = \frac{e^{-2\gamma\tau}(\theta_z + T_\beta^{I2}\theta_\epsilon + \theta_\varphi)}{2\theta_\epsilon T^I_\beta}$. This induces risk perceptions $\hat{\beta}^I = T^I_\beta < \hat{\beta}^U = T^U_{\beta f}$.

b. If τ is relatively high with $\tau \in [T_{\tau 1}, T_\tau]$, the firm discloses $\forall \beta_L \leq \beta$, and $\beta^*_H = \beta$. This induces risk perceptions $\hat{\beta}^I = \hat{\beta}^U = \beta$.

Define $\tilde{T}^U_\beta \equiv \frac{e^{-2\gamma\tau}(\theta_z + T_\beta^{I2}\theta_\epsilon + \theta_\varphi)}{2\theta_\epsilon T^I_\beta}$. The firm's optimal risk disclosure policy under low cost of information acquisition $\tau \leq T_\tau$ is summarized in Figure 3.6.

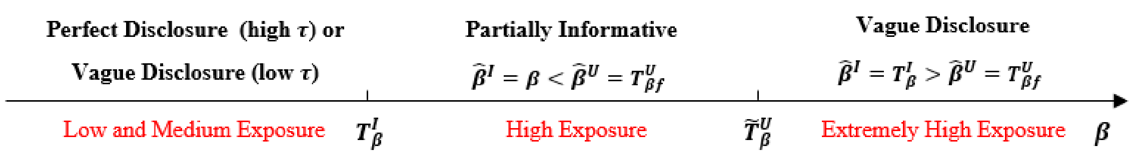

Figure 3.6: Risk Disclosure under Low Information Cost ($\tau \leq T_\tau$)

To compare the results under Proposition 6 and Proposition 7, note that the trough of informed investors T^I_β are the same in both cases. Under the case of high cost $\tau > T_\tau$, the area below T^I_β is further split by $\psi(\tau)\sqrt{\frac{\theta_z + \theta_\varphi}{\theta_\epsilon}}$, whereas under low cost $\tau \leq T_\tau$, the area above T^I_β is split by $\tilde{T}^U_\beta$. In total, the whole spectrum of β is split into four areas by $\tilde{T}^U_\beta$, T^I_β, and $\psi(\tau)\sqrt{\frac{\theta_z + \theta_\varphi}{\theta_\epsilon}}$ from high to low. Also remember that the ideal case for the firm is to guide informed investors' risk perceptions, but to indulge uninformed investors' own perceptions, because once unguided, the informed (uninformed) investors will form perceptions in a way that harms (benefits) the firm. However, whether these two goals could be simultaneously achieved depends on β.

First, for firms with extremely high exposure $\beta \geq \tilde{T}_{\beta}^{U}$, they would always provide vague risk disclosure regardless information acquisition cost τ. For such firms, both informed and uninformed tend to form lower perceptions given no disclosure, because the *learning effect* of perception in decreasing uncertainty dominates the *risk-bearing effect*. This implies that once given a disclosed interval, all investors tend to take the lower bound as perceptions (until certain troughs are met). When both groups form the same perception, the firm can not separately affect the two groups of investors, and prefers to be perceived with lower exposure, which is aligned with investors' natural tendency to form low perception given no disclosure. As a result, the firm has little incentive to use risk disclosure to *correct* investors' natural tendency, and hence would provide uninformative risk disclosure.

Second, for firms with high exposure $\beta \in [T_{\beta}^{I}, \tilde{T}_{\beta}^{U})$, they would provide partially informative risk disclosure when the cost τ is low enough, or disclose vaguely when the cost is high. For such firms, investors may not always perceive them at lower exposure as in the first case. Instead, when the cost τ is low enough, uninformed investors tend to form a higher perception but the informed tend to form lower perception. This implies that firms can separately affect the two groups of investors, using β_L to affect the informed, and β_H to affect the uninformed. Consequently, the two goals in the ideal case can be simultaneously achieved: $\beta_L = \beta$ to guide the informed investors, but $\forall \beta_H \geq T_{\beta f}^{U}$ to un-guide the uninformed. However, when the cost τ is high, things get back to the first case. Similar rationale applies to the case when firms have medium exposure $\beta \in [\sqrt{\frac{\theta_z+\theta_\varphi}{\theta_\epsilon}}, T_{\beta}^{I})$, and low exposure $\beta < \sqrt{\frac{\theta_z+\theta_\varphi}{\theta_\epsilon}}$.

The comparison of Proposition 6 and 7 shows that lowering the cost of information acquisition may have adverse effects. Conditional on a range of *high* cost $\tau > T_\tau$, decreasing the cost to $\tau \in (T_{\tau 1}, T_\tau]$ improves the quality of risk disclosure: the medium exposure firms shift from partially informative to perfect disclosure, and the high exposure firms shift from vague to partially informative disclosure, with firms of extremely low or high exposure unchanged. However, conditional on a range of *low* cost $\tau \in (T_{\tau 1}, T_\tau]$, further decreasing the cost to $\tau < T_{\tau 1}$ would worsen the quality of risk disclosure, as both the low and medium exposure firms shift from perfect to vague disclosure. This is intuitive because when acquisition cost is very low, there is little need for the firm to induce extra *perceived* incentive for investors to learn. Finally, this implies that risk disclosure quality is a hump-shaped function of the information acquisition cost.

To conclude, the results under the relaxed assumption of informed investors' risk perception choice are consistent with those under the restricted case as long as the information acquisition cost is not extremely low. In this case, less exposed firms tend to disclose more precisely in order to encourage information acquisition about the risk. Particularly the least exposed firms would provide perfect risk disclosure. However, when the information acquisition cost is very low, firms would *never* provide perfect risk disclosure.

Chapter 4

Further Discussion

4.1 Main Drivers of Results

There are four important elements in the model: disclosure softness, ambiguity aversion, information acquisition, and feedback effect. While the comparison of the benchmark and the main model demonstrates the effects of information acquisition and feedback effect, more analyses are needed to further disentangle the effects of each element on the results.

First, consider the effects of disclosure softness in the special case in which the firm discloses only one level of exposure β_d. All investors, informed or uninformed, just take the risk disclosure at the face value, and set their perceptions at β_d. In this case, the firm faces an additional constraint of equating $\hat{\beta}^I = \hat{\beta}^U$ when it solves the same problems in the main model, rendering the optimization results no better than the main model. As Lemma 1 shows, price informativeness degenerates to a monotonically decreasing function of risk perception when $\hat{\beta}^I = \hat{\beta}^U$. The firm therefore will always disclose $\beta_d \to 0$, rendering no information acquisition in equilibrium. To conclude, the softness of risk disclosure gives the firm more flexibility in promoting market learning.

Second, the results in the main model would generally flip if investors are ambiguity seeking instead of ambiguity averse, because now given a *U-shape* preference of risk perception, investors will choose their perceptions at the level that is *furthest* from the trough, rather than *closest* to the trough. However, the framework still work following the same rationale.

4.2 Policy Implications

The model has some policy implications.

First, requiring more precise risk disclosure may harm real efficiency, if the risks are unfamiliar to everyone, and about which firms can learn from the financial markets. Examples include cybersecurity risks, environmental risks, among other risks that are relatively new to our society. For such risks, it is sometimes optimal for firms *not* to provide precise risk disclosure to elicit information acquisition by investors, which helps firms' future decisions and hence improves economic efficiency.

Second, the results show that firms with high exposure to common risks provide less precise risk disclosures. To the extent that the purpose to mandate risk disclosures is to reveal to investors the most exposed entities in the economy, the model shows that mandatory risk disclosure may not always achieve this goal, at least in the realm of *qualitative* risk disclosure.

Finally, despite existing regulatory effort such as the Reg FD to preclude selective disclosures by firms, the model shows the possibility for firms to strategically design risk disclosures in a way that the same financial report could induce different risk perceptions among investors in an anticipated way. More generally, if firms understand the pattern how individual attributes explain investors' processing of soft disclosures[1], firms may be able to tailor their language to elicit specific perceptions among specific groups of financial report users, which is to some extent selective disclosures. Much more research is needed to understand the implications of this

[1] In the paper, the investor attribute firms take into consideration is information endowment. In real life, many other attributes such as financial literacy may affect how individuals process soft information.

problem.

4.3 Empirical Applications

The model generates several testable predictions.

First, the model sheds light on the relation of firms' risk exposure and the quality of risk disclosure. In particular, Proposition 4, Proposition 6 and Proposition 7 predict that firms with lower exposure provide more precise risk disclosures, when the cost of information acquisition is not extremely low. If the exposure is high enough, firms tend to provide completely uninformative disclosure.

The model also speaks to the relation of the cost of information acquisition and the quality of risk disclosure. In general, the risk disclosures of firms with moderate exposure are most sensitive to the cost, whereas the most (least) exposed firms tend to disclose vaguely (perfectly) as long as the cost is not extremely low. Besides, the average precision of risk disclosure is a hump-shaped function of the cost.

Chapter 5

Conclusion

This paper studies how firms decide *qualitative* risk disclosure to affect investors' information acquisition about common risks, when firms privately know their exposure to the risks. Based on a model with unknown payoff distribution, soft disclosure, and ambiguity averse investors, I find that to encourage information acquisition, firms with lower exposure generally provide more precise risk disclosures. In particular, low exposure firms tend to provide *perfect* risk disclosures, medium firms provide *partially informative* disclosures, whereas high exposure firms tend to disclose vaguely.

I show that the softness of risk disclosure enables firms to induce *different* risk perceptions among informed and uninformed investors, which implies more flexibility to affect investors' incentive to acquire private information. In particular, the incentive to acquire private information is stronger if informed investors have *lower* perceived uncertainty, and/or uninformed investors have *higher* perceived uncertainty. With the flexibility to induce two different perceptions $\hat{\beta}^I$ and $\hat{\beta}^U$, the firm can separately manage the uncertainty perceived by the informed and the uninformed, and hence enjoys more flexibility to shape the comparative advantage of getting informed.

When firms can exert separate influences on $\hat{\beta}^I$ and $\hat{\beta}^U$ with the same risk disclosure, it is optimal to guide the perceptions of the informed but to un-guide the uninformed, because conditional on being unguided (or equivalently given no risk disclosure), the informed (uninformed) investors would form perceptions in the way that firms dislike (prefer). However, whether firms have the flexibility to exert separate influences on the two groups of investors depends on the level of true risk exposure. Generally speaking, when true exposure is extremely high or extremely low, and hence falls on the same side of the two troughs of investors' preferences, both informed and uninformed investors tend to take the same bound of the disclosures as perceptions, implying that firms has less flexibility to exert separate influences, and therefore less flexibility to affect information acquisition.

The paper also explores the effects of the information acquisition cost. On one hand, lower information acquisition cost always enables firms to promote higher price informativeness, which improves economic efficiency. On the other hand, extremely low cost of information acquisition may impair the quality of risk disclosure, making it harder for investors to form correct risk perceptions. This indicates a potential trade-off between information efficiency and economic efficiency.

To conclude, this paper is only the initial attempt to study how firms may use disclosures to guide investors' information acquisition if they need to learn from the market. This is especially relevant in cases when firms need to understand low-frequency external risks for better risk management, investment or other major business decisions. Many interesting questions are open to future research.

Appendix A

Proofs

A.1 Proof of Proposition 2

Ambiguity averse investors solve the max-min problem below:

$$\max_{x_j} \quad \min_{\hat{\beta}} \quad E(v - p|I_j)x_j - \frac{\gamma Var(v|I_j)}{2} x_j^2$$

Given the expression of $E(v|I_j)$ and $Var(v|I_j)$, the minimizing problem to determine $\hat{\beta}$ given x_j is:

$$\min_{\hat{\beta}} \quad (v_1 - p)x_j - \frac{\gamma \hat{\beta}}{2(\theta_z + \hat{\beta}^2 \theta_\epsilon)} x_j^2$$

Take the first derivative with respect to $\hat{\beta}$:

$$\frac{\gamma x_j^2 (\hat{\beta}^2 \theta_\epsilon - \theta_z)}{2(\theta_z + \hat{\beta}^2 \theta_\epsilon)^2} \begin{cases} \geq 0, & \hat{\beta} \geq \frac{\sigma_\epsilon}{\sigma_z} \\ < 0, & \hat{\beta} < \frac{\sigma_\epsilon}{\sigma_z} \end{cases}$$

So the optimal perceived exposure of ambiguity averse investors is:

$$\hat{\beta}^* = \begin{cases} \beta_L, & \beta_L \geq \frac{\sigma_\epsilon}{\sigma_z} \\ \frac{\sigma_\epsilon}{\sigma_z}, & \frac{\sigma_\epsilon}{\sigma_z} \in (\beta_L, \beta_H) \\ \beta_H, & \beta_H \leq \frac{\sigma_\epsilon}{\sigma_z} \end{cases}$$

Since the choice of $\hat{\beta}^*$ is independent of the demand choice x_j, there is no *inertia* equilibrium here in which $x_j^* = 0$ for any signals. Given $\hat{\beta}^*$, the optimal demand is:

$$x_j^* = \frac{(v_1 - p)(\theta_z + \hat{\beta^*}^2 \theta_\epsilon)}{\gamma \hat{\beta}^*}$$

A.2 Proof of Proposition 4

Note that the troughs satisfy $T_\beta^U = T_{\beta f}^U < T_\beta^I = T_{\beta f}^I$. Discuss in three cases.

A.2.1 Real Exposure $\beta \geq T_\beta^I$

In this case, β_H must exceed T_β^I, so focus on β_L. Given investors' strategies, if $\beta_L \geq T_\beta^I$ (denoted Choice A), $\hat{\beta}^I = \hat{\beta}^U = \beta_L$. Then firms' problem becomes:

$$\max_{\beta_L} \quad \theta_p = e^{-2\gamma\tau}(\theta_z + \beta_L^2 \theta_\epsilon + \theta_\varphi) - \theta_z - \beta_L^2 \theta_\epsilon$$

Because $e^{-2\gamma\tau} < 1$, θ_p decreases with β_L. So the optimal lower bound is set as low as possible at $\beta_L = T_\beta^I$, rendering $\hat{\beta}^I = \hat{\beta}^U = T_\beta^I$.

Next if $\beta_L \in [T_\beta^U, T_\beta^I)$ (denoted Choice B), $\hat{\beta}^I = T_\beta^I > \hat{\beta}^U = \beta_L$. Because θ_p is an inverse U-shape function of $\hat{\beta}^U$ with peak at $T_{\beta f}^U = \frac{e^{-2\gamma\tau}(\theta_z + \hat{\beta^I}^2 \theta_\epsilon + \theta_\varphi)}{2\theta_\epsilon \hat{\beta}^I}$, the optimal lower bound would be $\beta_L^* = T_{\beta f}^U = \frac{e^{-2\gamma\tau}(\theta_z + T_\beta^{I2} \theta_\epsilon + \theta_\varphi)}{2\theta_\epsilon T_\beta^I} = e^{-2\gamma\tau}\sqrt{\frac{\theta_z + \theta_\varphi}{\theta_\epsilon}}$. Moreover, further lowering β_L below $T_{\beta f}^U$ yields no improvement.

In conclusion, when firms' real exposure $\beta \geq T_\beta^I$, the optimal risk disclosure is $\forall \beta_H \geq \beta$, and $\forall \beta_L \leq T_{\beta f}^U = e^{-2\gamma\tau} T_\beta^I$, rendering investors perceptions as $\hat{\beta}^I = T_\beta^I > \hat{\beta}^U = e^{-2\gamma\tau} T_\beta^I$.

A.2.2 Real Exposure $\beta \leq T_\beta^U$

In this case, β_L must fall below T_β^U, so focus on β_H. Given investors' strategies, if $\beta_H \leq T_\beta^U$ (denoted Choice A), $\hat{\beta}^I = \hat{\beta}^U = \beta_H$. Similar to the first case, the optimal upper bound is as low as possible at $\beta_H = \beta$.

Next if $\beta_H \in (T_\beta^U, T_\beta^I]$, $\hat{\beta}^I = \beta_H > \hat{\beta}^U = T_\beta^U$. To induce $\hat{\beta}^i$ further away from T_β^I, it is optimal to set $\beta_H^* = T_\beta^U$, rendering $\hat{\beta}^I = \hat{\beta}^U$ again. However, this is not better than Choice A, because when the two perceptions are the same, lower perception is better. Finally, increasing β_H above T_β^I is no better.

In conclusion, when firms' real exposure $\beta \leq T_\beta^I$, the optimal risk disclosure is $\forall \beta_L \leq \beta, \beta_H^* = \beta$, rendering investors perceptions as $\hat{\beta}^I = \hat{\beta}^U = \beta$. This implies that firms would disclose perfectly in this case. The equilibrium price informativeness is $\theta_p^* = (e^{-2\gamma\tau} - 1)(\theta_z + \beta^2\theta_\epsilon) + e^{-2\gamma\tau}\theta_\varphi$. Moreover, to ensure that the assumption $\beta \leq T_\beta^U$ holds, it is necessary to have:

$$T_\beta^U = \frac{e^{-2\gamma\tau}(\theta_z + \beta^2\theta_\epsilon + \theta_\varphi)}{2\theta_\epsilon \beta} \leq \beta$$

which yields $\beta \leq \psi(\tau)\sqrt{\frac{\theta_z + \theta_\varphi}{\theta_\epsilon}} = \psi(\tau) T_\beta^I$, with $\psi(\tau) \equiv \frac{1}{2e^{2\gamma\tau} - 1}$.

A.2.3 Real Exposure $\beta \in (T_\beta^U, T_\beta^I)$

If $T_\beta^U \leq \beta_L \leq \beta_H \leq T_\beta^I$, $\hat{\beta}^I = \beta_H \geq \hat{\beta}^U = \beta_L$. Similarly, the firm prefers $\hat{\beta}^I$ to be further away from T_β^I as low as possible, rendering $\beta_H^* = \beta$, and prefers $\hat{\beta}^U$ to be close to $T_{\beta f}^U$. Therefore, $\beta_L^* = T_{\beta f}^U = \frac{e^{-2\gamma\tau}(\theta_z + \beta^2\theta_\epsilon + \theta_\varphi)}{2\theta_\epsilon \beta}$.

It is easy to verify that neither decreasing β_L below T_β^U, or increasing β_H above T_β^I would bring any improvement. In conclusion, when the firm's real exposure falls in $\beta \in (T_\beta^U, T_\beta^I)$, the optimal upper bound is $\beta_H^* = \beta$. The optimal disclosure is

$\forall \beta_L \leq T^U_{\beta f} = \frac{e^{-2\gamma\tau}(\theta_z+\beta^2\theta_\epsilon+\theta_\varphi)}{2\theta_\epsilon\beta}$, and $\beta^*_H = \beta$, which leads to risk perceptions of $\hat{\beta}^I = \beta > \hat{\beta}^U = T^U_{\beta f}$. To ensure that the assumption $\beta > T^U_\beta$ holds, it is necessary to have $\beta > \psi(\tau)T^I_\beta$.

A.3 Proof of Proposition 5

Informed ambiguity averse investors solve the max-min problem:

$$\max_{x^I_j} \ \min_{\hat{\beta}^I} \ E(v-p|I^I_j)x^I_j - \frac{\gamma Var(v|I^I_j)}{2}(x^I_j)^2$$

Based on the law of total variance, $Var(v|I^I_j) = \hat{\beta}^I Var(z|v_1,u,p,s_z)$. Define $Y \equiv z - a^*$. Given any x^I_j, the minimizing problem to determine $\hat{\beta}^I$ is:

$$\min_{\hat{\beta}^I} \quad (v_1-p)\,x^I_j - \sqrt{\hat{\beta}^I}E(Y|I^I)x^I_j - \frac{\gamma\hat{\beta}^I}{2(\theta_z+\hat{\beta}^{I^2}\theta_\epsilon+\theta_\varphi)}(x^I_j)^2$$

Things get complicated here as $E(Y|I^I) \neq 0$. Instead $E(Y|I^I)$ is now similar to a weighted average of all available signals v_1, s_z and $\tilde{p}$, with weights being functions of $\hat{\beta}^I$. $E(Y|I^I)$ can be expressed as:

$$E(Y|I^I) = -g_1 v_1 + g_2 s_z - g_3\tilde{p} - g_4\bar{z}$$

with coefficients:

$$\begin{cases} g_1 = \hat{\beta}^I \theta_\epsilon \left(\frac{1}{\theta_z + \hat{\beta^I}^2 \theta_\epsilon + \theta_p} - \frac{1}{\theta_z + \hat{\beta^I}^2 \theta_\epsilon + \theta_\varphi} \right) \\ g_2 = \frac{\theta_\varphi}{\theta_z + \hat{\beta^I}^2 \theta_\epsilon + \theta_\varphi} \\ g_3 = \frac{\theta_p}{\theta_z + \hat{\beta^I}^2 \theta_\epsilon + \theta_p} \\ g_4 = \theta_z \left(\frac{1}{\theta_z + \hat{\beta^I}^2 \theta_\epsilon + \theta_p} - \frac{1}{\theta_z + \hat{\beta^I}^2 \theta_\epsilon + \theta_\varphi} \right) \end{cases}$$

So it is equivalent to solve:

$$\min_{\hat{\beta}^I} \quad (v_1 - p)\, x_j^I + \sqrt{\hat{\beta}^I}\, [g_1 v_1 + g_2(-s_z) + g_3 \tilde{p} + g_4 \bar{z}]\, x_j^I - \frac{\gamma \hat{\beta}^I}{2(\theta_z + \hat{\beta^I}^2 \theta_\epsilon + \theta_\varphi)} (x_j^I)^2$$

This objective function is too complicated a function of $\hat{\beta}^I$. To simplify the problem, consider the case in which θ_z and θ_ϵ are significantly smaller than $\theta_\varphi \gg 0$, implying that the risk factor is highly uncertain and market learning about the common risk factor is highly effective (This is exactly when feedback effect is important). Under these conditions, g_1 and g_4 would be close to zero, while g_2 and g_3 are close to one. So the problem becomes:

$$\min_{\hat{\beta}^I} \quad \left[v_1 - p + \sqrt{\hat{\beta}^I}\, (\tilde{p} - s_z) \right] x_j^I - \frac{\gamma \hat{\beta}^I}{2(\theta_z + \hat{\beta^I}^2 \theta_\epsilon + \theta_\varphi)} (x_j^I)^2$$

Next is to approximate the objective function under different x_j^I. To avoid the unnecessary complication of inertia equilibrium, I assume that when informed investors choose perception, they act *as if* they have already decided whether to buy ($x_j^I \geq 0$) or sell ($x_j^I < 0$) based on the sign of $v_1 - p$. They consider the magnitude of $\sqrt{\hat{\beta}^I}\, (\tilde{p} - s_z)$ only to the extent how much it increases the magnitude of the order to buy (sell) when $v_1 - p$ is positive (negative).

Suppose $v_1 \geq p$ and hence $x_j^I \geq 0$. Then the objective function can be approximated by:

$$\min_{\hat{\beta}^I} \quad (v_1 - p)\, x_j^I + \sqrt{\hat{\beta}^I}\, [(\tilde{p} - s_z) * \mathbb{1}(\tilde{p} - s_z \geq 0)]\, x_j^I - \frac{\gamma \hat{\beta}^I}{2(\theta_z + {\hat{\beta}^I}^2 \theta_\epsilon + \theta_\varphi)} (x_j^I)^2$$

Considering the variance effect only, informed investors would choose $\hat{\beta}^I$ as close as possible to the trough $\sqrt{\frac{\theta_z + \theta_\varphi}{\theta_\epsilon}}$, which is very similar to the benchmark case. However, given the potential positive level effect for a long position $x_j \geq 0$, the trough would be lower as informed investors have an extra incentive to decrease $\hat{\beta}^I$ in order to limit the positive level effect. Specifically, the first derivative with respect to $\hat{\beta}^I$ is:

$$\begin{aligned} & \frac{1}{2\sqrt{\hat{\beta}^I}} [(\tilde{p} - s_z) * \mathbb{1}(\tilde{p} - s_z \geq 0)]\, x_j^I - \frac{\gamma(\theta_z + \theta_\varphi - {\hat{\beta}^I}^2 \theta_\epsilon)}{2(\theta_z + {\hat{\beta}^I}^2 \theta_\epsilon + \theta_\varphi)^2} (x_j^I)^2 \\ \equiv \; & \kappa_1 \frac{1}{\sqrt{\hat{\beta}^I}} + \kappa_2 ({\hat{\beta}^I}^2 \theta_\epsilon - \theta_z - \theta_\varphi) \end{aligned}$$

where $\kappa_1 \geq 0$ and $\kappa_2 > 0$. Informed investors' utility is still a *U-shape* function of $\hat{\beta}^I$ with a trough at $T_\beta^I < \sqrt{\frac{\theta_z + \theta_\varphi}{\theta_\epsilon}}$.

Repeat the same procedure in the case of $x_j^I < 0$. Since x_j^I is negative, the original objective function can be approximated by:

$$\min_{\hat{\beta}^I} \quad (v_1 - p)\, x_j^I + \sqrt{\hat{\beta}^I}\, [(\tilde{p} - s_z) * \mathbb{1}(\tilde{p} - s_z < 0)]\, x_j^I - \frac{\gamma \hat{\beta}^I}{2(\theta_z + {\hat{\beta}^I}^2 \theta_\epsilon + \theta_\varphi)} (x_j^I)^2$$

As a result, the first order condition becomes:

$$
\begin{aligned}
&\frac{1}{2\sqrt{\hat{\beta}^I}}\left[(\tilde{p}-s_z) * \mathbb{1}(\tilde{p}-s_z<0)\right] x_j^I - \frac{\gamma(\theta_z+\theta_\varphi-\hat{\beta^I}^2\theta_\epsilon)}{2(\theta_z+\hat{\beta^I}^2\theta_\epsilon+\theta_\varphi)^2}(x_j^I)^2 \\
\equiv\ &\kappa_3\frac{1}{\sqrt{\hat{\beta}^I}} - \kappa_4(\theta_z+\theta_\varphi-\hat{\beta^I}^2\theta_\epsilon)
\end{aligned}
$$

where $\kappa_3 \geq 0$ and $\kappa_4 > 0$. Again, informed investors' utility is a *U-shape* function of $\hat{\beta}^I$ with a trough at $T_\beta^I < \sqrt{\frac{\theta_z+\theta_\varphi}{\theta_\epsilon}}$.

Now we can see that the sign of the derivative under $x_j^I < 0$ follows the same pattern as in the $x_j^I \geq 0$ case. Consequently, informed investors choose perceived exposure in the same way regardless of the sign of x_j^I. Therefore, assuming low θ_z and θ_ϵ, and high $\theta_\varphi \gg 0$, the optimal perceived risk exposure of informed investors is:

$$
\hat{\beta}^I = \begin{cases} \beta_L, & \beta_L \geq T_\beta^I \\ T_\beta^I, & T_\beta^I \in (\beta_L, \beta_H) \\ \beta_H, & \beta_H \leq T_\beta^I \end{cases}
$$

where $T_\beta^I < \sqrt{\frac{\theta_z+\theta_\varphi}{\theta_\epsilon}}$. Because the choice of $\hat{\beta}^I$ imposes no constraint on x_j^I, *inertia* does not exist. Given $\hat{\beta}^I$, the optimal demand of informed investors is:

$$
x_j^I = \frac{\left[v_1 - p - \sqrt{\hat{\beta}^I}E(Y|I^I)\right](\theta_z+\hat{\beta^I}^2\theta_\epsilon+\theta_\varphi)}{\gamma\hat{\beta}^I}
$$

For uninformed investors, $E(Y|I^U)=0$, and $Var(Y|I^U) = \hat{\beta}^U Var(z|v_1,u,p) = \frac{\hat{\beta}^U}{\theta_z+\hat{\beta^U}^2\theta_\epsilon+\theta_p}$. Besides, their risk perception $\hat{\beta}^U$ does not affect price informativeness

(Expression 3.16). Go through the same process above will yield similar result:

$$\hat{\beta^U} = \begin{cases} \beta_L, & \beta_L \geq T^U_\beta \\ T^U_\beta, & T^U_\beta \in (\beta_L, \beta_H) \\ \beta_H & \beta_H \leq T^U_\beta \end{cases}$$

where $T^U_\beta = \sqrt{\frac{\theta_z+\theta_p}{\theta_\epsilon}}$. Given $\hat{\beta^U}$, the optimal demand of uninformed investors is:

$$x^U_j = \frac{(v_1 - p)\left(\theta_z + \hat{\beta^U}^2\theta_\epsilon + \theta_p\right)}{\gamma\hat{\beta^U}}$$

A.4 Proof of Proposition 6

Under high cost of information acquisition with $\tau > T_\tau$, the troughs satisfy $T^U_{\beta f} = T^U_\beta < T^I_\beta < T^I_{\beta f}$.

Real Exposure $\beta \geq T^I_\beta$

In this case, β_H must exceed T^I_β, so focus on β_L. Given investors' strategies, if $\beta_L \geq T^I_\beta$ (denoted Choice A), $\hat{\beta^I} = \hat{\beta^U} = \beta_L$. Then the firm's problem becomes:

$$\max_{\beta_L} \quad \theta_p = e^{-2\gamma\tau}(\theta_z + \beta_L^2\theta_\epsilon + \theta_\varphi) - \theta_z - \beta_L^2\theta_\epsilon$$

Because $e^{-2\gamma\tau} < 1$, θ_p decreases with β_L. So the optimal lower bound is set as low as possible at $\beta_L = T^I_\beta$, rendering $\hat{\beta^I} = \hat{\beta^U} = T^I_\beta$.

Next if $\beta_L \in [T^U_\beta, T^I_\beta)$ (denoted Choice B), $\hat{\beta^I} = T^I_\beta > \hat{\beta^U} = \beta_L$. Because θ_p is an inverse U-shape function of $\hat{\beta^U}$ with peak at $T^U_{\beta f} = \frac{e^{-2\gamma\tau}(\theta_z+\hat{\beta^I}^2\theta_\epsilon+\theta_\varphi)}{2\theta_\epsilon\hat{\beta^I}}$, the optimal lower bound would be $\beta^*_L = T^U_{\beta f} = \frac{e^{-2\gamma\tau}(\theta_z+T^{I2}_\beta\theta_\epsilon+\theta_\varphi)}{2\theta_\epsilon T^I_\beta}$. Moreover, further lowering β_L

below $T^U_{\beta f}$ yields no improvement.

In conclusion, when firms' real exposure $\beta \geq T^I_\beta$, the optimal risk disclosure is $\forall \beta_H \geq \beta$, and $\forall \beta_L \leq T^U_{\beta f} = \frac{e^{-2\gamma\tau}(\theta_z + T^{I2}_\beta \theta_\epsilon + \theta_\varphi)}{2\theta_\epsilon T^I_\beta}$, rendering investors perceptions as $\hat{\beta}^I = T^I_\beta > \hat{\beta}^U = T^U_{\beta f}$.

A.4.1 Real Exposure $\beta \leq T^U_\beta$

In this case, β_L must fall below T^U_β, so focus on β_H. Given investors' strategies, if $\beta_H \leq T^U_\beta$ (denoted Choice A), $\hat{\beta}^I = \hat{\beta}^U = \beta_H$. Similar to the first case, the optimal upper bound is as low as possible at $\beta_H = \beta$.

Next if $\beta_H \in (T^U_\beta, T^I_\beta]$, $\hat{\beta}^I = \beta_H > \hat{\beta}^U = T^U_\beta$. The choice of β_H depends on $\frac{\partial \theta_p}{\partial \hat{\beta}^I}$. Since θ_p is a U-shape function of $\hat{\beta}^I$ with the trough at $\sqrt{\frac{\theta_z + \theta_\varphi}{\theta_\epsilon}}$, which is higher than T^I_β. Therefore, $\frac{\partial \theta_p}{\partial \hat{\beta}^I} < 0$ in this range of $(T^U_\beta, T^I_\beta]$, rendering $\beta^*_H = T^U_\beta$. However, this is not better than Choice A. Finally, increasing β_H above T^I_β is no better. Therefore the optimal choice is Choice A, rendering the equilibrium price informativeness as $\theta^*_p = (e^{-2\gamma\tau} - 1)(\theta_z + \beta^2 \theta_\epsilon) + e^{-2\gamma\tau}\theta_\varphi$.

In conclusion, when firms' real exposure $\beta \leq T^I_\beta$, the optimal risk disclosure is $\forall \beta_L \leq \beta, \beta^*_H = \beta$, rendering investors perceptions as $\hat{\beta}^I = \hat{\beta}^U = \beta$. This implies that firms would disclose perfectly in this case. In addition, to ensure that the assumption $\beta \leq T^U_\beta$ holds, it is necessary that $\beta \leq \psi(\tau)\sqrt{\frac{\theta_z + \theta_\varphi}{\theta_\epsilon}}$, with $\psi(\tau) \equiv \sqrt{\frac{1}{2e^{2\gamma\tau} - 1}}$.

A.4.2 Real Exposure $\beta \in (T^U_\beta, T^I_\beta)$

If $T^U_\beta \leq \beta_L \leq \beta_H \leq T^I_\beta$, $\hat{\beta}^I = \beta_H \geq \hat{\beta}^U = \beta_L$. Similarly, the firm prefers $\hat{\beta}^I$ to be further away from T^I_β as low as possible, rendering $\beta^*_H = \beta$, and prefers $\hat{\beta}^U$ to be close to $T^U_{\beta f}$. Therefore, $\beta^*_L = T^U_{\beta f} = \frac{e^{-2\gamma\tau}(\theta_z + \beta^2 \theta_\epsilon + \theta_\varphi)}{2\theta_\epsilon \beta}$.

It is easy to verify that neither decreasing β_L below T^U_β, or increasing β_H above

T_β^I would bring any improvement. In conclusion, when the firm's real exposure falls in $\beta \in (T_\beta^U, T_\beta^I)$, the optimal upper bound is $\beta_H^* = \beta$. The optimal disclosure is $\beta_L^* = T_{\beta f}^U = \frac{e^{-2\gamma\tau}(\theta_z+\beta^2\theta_\epsilon+\theta_\varphi)}{2\theta_\epsilon\beta}$, and $\beta_H^* = \beta$, which leads to risk perceptions of $\hat{\beta}^I = \beta > \hat{\beta}^U = T_{\beta f}^U$. To ensure that the assumption $\beta \leq T_\beta^U$ holds, it is necessary that $\beta > \psi(\tau)\sqrt{\frac{\theta_z+\theta_\varphi}{\theta_\epsilon}}$

A.5 Proof of Proposition 7

Under low cost of information acquisition with $\tau \leq T_\tau$, the troughs satisfy $T_{\beta f}^I > T_{\beta f}^U = T_\beta^U \geq T_\beta^I$.

A.5.1 Real Exposure $\beta \geq T_\beta^U$

In this case, β_H must exceed T_β^U, so focus on β_L. Given investors' strategies, if $\beta_L \geq T_\beta^U$ (denoted Choice A), $\hat{\beta}^I = \hat{\beta}^U = \beta_L$. Then firms' problem becomes:

$$\max_{\beta_L} \quad \theta_p = e^{-2\gamma\tau}(\theta_z + \beta_L^2\theta_\epsilon + \theta_\varphi) - \theta_z - \beta_L^2\theta_\epsilon$$

Because $e^{-2\gamma\tau} < 1$, θ_p decreases with β_L. So the optimal lower bound is $\beta_L = T_\beta^U$. Next if $\beta_L \in [T_\beta^I, T_\beta^U)$, risk perceptions would be $\hat{\beta}^I = \beta_L < \hat{\beta}^U = T_\beta^U$ (denoted Choice B). Since firms prefer $\hat{\beta}^I$ deviating from $T_{\beta f}^I$, $\beta_L = T_\beta^I$. This yields better result than Choice A. Finally if $\beta_L < T_\beta^I$ (denoted Choice C), $\hat{\beta}^I = T_\beta^I < \hat{\beta}^U = T_\beta^U$, which is not better than Choice B.

In conclusion, when firms' real exposure $\beta \geq T_\beta^U$, the optimal risk disclosure is $\forall \beta_L \leq T_\beta^I$, and $\forall \beta_H \geq \beta$. This induces risk perceptions of $\hat{\beta}^I = T_\beta^I < \hat{\beta}^U = T_\beta^U = \frac{e^{-2\gamma\tau}(\theta_z+T_\beta^{I2}\theta_\epsilon+\theta_\varphi)}{2\theta_\epsilon T_\beta^I}$. Besides, to satisfy the assumption $\beta \geq T_\beta^U$, it is necessary that $\beta \geq \frac{e^{-2\gamma\tau}(\theta_z+T_\beta^{I2}\theta_\epsilon+\theta_\varphi)}{2\theta_\epsilon T_\beta^I}$.

A.5.2 Real Exposure $\beta \leq T_\beta^I$

In this case, β_L must fall below T_β^I, so focus on β_H. Given investors' strategies, if $\beta_H \leq T_\beta^I$ (denoted Choice A), $\hat{\beta^I} = \hat{\beta^U} = \beta_H$. Since the firm prefers lower perception in this case, the optimal upper bound is $\beta_H = \beta$. Next if $\beta_H \in (T_\beta^I, T_\beta^U]$, $\hat{\beta^I} = T_\beta^I < \hat{\beta^U} = \beta_H$. To get closer to $T_{\beta f}^U$, $\beta_H^* = T_{\beta f}^U = \frac{e^{-2\gamma\tau}(\theta_z + T_\beta^{I2}\theta_\epsilon + \theta_\varphi)}{2\theta_\epsilon T_\beta^I}$ (Choice B). The resultant price informativeness is $\theta_p(B) = \frac{e^{-4\gamma\tau}(\theta_z + T_\beta^{I2}\theta_\epsilon + \theta_\varphi)^2}{4\theta_\epsilon T_\beta^{I2}} - \theta_z$.

Compare with the price informativeness under Choice A $\theta_p(A) = e^{-2\gamma\tau}(\theta_z + \beta^2\theta_\epsilon + \theta_\varphi) - \theta_z - \beta^2\theta_\epsilon$:

$$\theta_p(B) - \theta_p(A) = \left[e^{-2\gamma\tau}(\theta_z + T_\beta^{I2}\theta_\epsilon + \theta_\varphi) - 2T_\beta^{I2}\theta_\epsilon\right]^2 - 4\left[(T_\beta^I\theta_\epsilon)^2(T_\beta^{I2} - \beta^2)(1 - e^{-2\gamma\tau})\right]$$

Define $y \equiv e^{-2\gamma\tau} \in (0, 1]$. The first term in brackets is a U-shape function of y with trough at $y^* = \frac{2T_\beta^{I2}\theta_\epsilon}{\theta_z + \beta^2\theta_\epsilon + \theta_\varphi} < 1$, which renders the first term zero. The second term in brackets is a decreasing function of y, which is smaller than the first term when $y = 0$, and equals to zero when $y = 1$. This implies that the two functions will intersect twice at two points, denoted by $y_1 = e^{-2\gamma T_{\tau 1}} > y^*$ and $y_2 = e^{-2\gamma T_{\tau 2}} < y^*$. However, given the primary assumption that the cost is low enough with $\tau \leq T_\tau$, and hence $T_\beta^U \geq T_\beta^I$, we only need to consider the area with $y \geq y^* = \frac{2T_\beta^{I2}\theta_\epsilon}{\theta_z + \beta^2\theta_\epsilon + \theta_\varphi}$. In this area, the two functions only intersects once at $y_1 > y^*$. Consequently, if $y > y_1$ under low enough information cost, i.e., $\forall\tau < T_{\tau 1}$, Choice B would be better than Choice A, whereas for $\forall\tau \in [T_{\tau 1}, T_\tau]$, Choice A dominates. Finally, increasing β_H above T_β^U is not better than Choice B.

In conclusion, when the firm's real exposure $\beta \leq T_\beta^I$, it is optimal to disclose $\forall\beta_L \leq \beta$, and $\beta_H^* = \beta$, if the information cost is low enough. Otherwise, it is optimal to set $\forall\beta_L \leq \beta$, and $\forall\beta_H \geq T_{\beta f}^U = \frac{e^{-2\gamma\tau}(\theta_z + T_\beta^{I2}\theta_\epsilon + \theta_\varphi)}{2\theta_\epsilon T_\beta^I}$.

A.5.3 Real Exposure $\beta \in (T_\beta^I, T_\beta^U)$

If $T_\beta^I \leq \beta_L \leq \beta_H \leq T_\beta^U$, $\hat{\beta^I} = \beta_L < \hat{\beta^U} = \beta_H$. Similarly, the firm prefers $\hat{\beta^I}$ to be further away from T_β^I as low as possible, rendering $\beta_L^* = \beta$, and prefers $\hat{\beta^U}$ to be close to $T_{\beta f}^U$, rendering $\forall \beta_H \geq T_{\beta f}^U = \frac{e^{-2\gamma\tau}(\theta_z + \beta^2\theta_\epsilon + \theta_\varphi)}{2\theta_\epsilon \beta}$ the best choice.

In addition, neither decreasing β_L below T_β^I nor increasing β_H above T_β^U would bring improvement. In conclusion, when the firm's real exposure falls in $\beta \in (T_\beta^I, T_\beta^U)$, it is optimal to set $\beta_L^* = \beta$, and $\forall \beta_H \geq T_{\beta f}^U = \frac{e^{-2\gamma\tau}(\theta_z + \beta^2\theta_\epsilon + \theta_\varphi)}{2\theta_\epsilon \beta}$. Besides, to satisfy the assumption $\beta \geq T_\beta^U$, it is necessary that $\beta < \frac{e^{-2\gamma\tau}(\theta_z + T_\beta^{I2}\theta_\epsilon + \theta_\varphi)}{2\theta_\epsilon T_\beta^I}$.

Appendix B

Benchmark Case Revisited

In this section, I revisit the benchmark case if the firm cares about stock price instead of firm value. The firm's risk management action at $t = 5$ and investors' decisions at $t = 3$ are the same, with a different disclosure policy to maximize its stock price based utility.

Given investors' optimal choice of risk perception and stock demand, the firm sets its risk disclosure policy $[\beta_L, \beta_H]$ at $t = 2$. Because investors have no private information about z, the firm could not learn about z from price, leaving no role for risk disclosure to change price informativeness. In addition, firm value is determined by real exposure β, not the perceived level $\hat{\beta}$. Therefore, *the firm cares about risk disclosure only if it cares about stock price.* Before solving directly the optimal $[\beta_L, \beta_H]$, I first solve for the firm's most preferred risk perception. For the firm, it maximizes its expected utility (averaging over q):

$$\begin{aligned} \max_{\hat{\beta}} E_q(U_f|I^f) &= E_q(p|I^f) - \frac{\gamma_f Var_q(p|I^f)}{2} \\ &= \left(e - \frac{\gamma_f \hat{\beta}}{\theta_z + \hat{\beta}^2\theta_\epsilon}\right) - \left(\frac{\gamma_f \hat{\beta}}{\theta_z + \hat{\beta}^2\theta_\epsilon}\right)^2 \sigma_q^2 \end{aligned} \tag{B.1}$$

The firm's expected utility always decreases with the ratio $\frac{\gamma_f \hat{\beta}}{\theta_z + \hat{\beta}^2\theta_\epsilon}$, which is the multiplier of the noise trading term in the stock price. Moreover, this multiplier turns out to be $\gamma Var(v|I_j)$. Therefore, maximizing the firm's expected utility is equivalent to minimizing the posterior variance of firm value $Var(v|I_j)$.

Lemma 2. *The posterior variance of firm value increases (decreases) with the perceived exposure $\hat{\beta}$ when perception is below (above) $\frac{\sigma_\epsilon}{\sigma_z}$. So the firm's expected utility is a U-shape function of $\hat{\beta}$ as:*

$$\frac{\partial E_q(U_f|I^f)}{\partial\hat{\beta}} \begin{cases} \leq 0, & \hat{\beta} \leq \frac{\sigma_\epsilon}{\sigma_z} \\ > 0, & \hat{\beta} > \frac{\sigma_\epsilon}{\sigma_z} \end{cases} \tag{B.2}$$

Lemma 2 shows that the expected utility of the firm is a U-shape function of the perceived risk exposure $\hat{\beta}$, implying that firms do not always desire to be regarded as bearing less common risk. Again, this is due to the two conflicting effects of $\hat{\beta}$ on the volatility of firm value: the *risk-bearing* effect and the *learning* effect.

Based on Lemma 2, if the lower bound of the disclosed interval β_L is set above the threshold $\frac{\sigma_\epsilon}{\sigma_z}$, the firm prefers a risk perception at β_H; If the upper bound of the interval β_H is below the threshold, the firm prefers a risk perception at β_L; If the threshold falls in the disclosed interval, the firm prefers to be perceived at either β_L or β_H. Furthermore, compare the threshold strategy of the firm and that of the investors. The thresholds in Expression (3.4) and (B.2) are the same, but the decision rule of the firm and the investors are different. The firm's optimal disclosure policy is shown in Proposition 8.

Proposition 8. *When the firm cares about the stock price, the optimal disclosure policy is:*

$$\begin{cases} \beta_L^* = \beta, \quad \forall\beta_H^* \geq \beta; \quad if\ \beta \geq \frac{\sigma_\epsilon}{\sigma_z} \\ \beta_H^* = \beta, \quad \forall\beta_L^* \leq \beta; \quad if\ \beta < \frac{\sigma_\epsilon}{\sigma_z} \end{cases} \tag{B.3}$$

Assuming that firms would be as precise as possible if they are indifferent, then Expression (B.3) implies that it is optimal for firms to provide perfect risk disclosure *for any value of true exposure, i.e.,* $\beta_L^* = \beta_H^* = \beta$ *for* $\forall\beta$.

Proposition 8 is easy to verify. If the firm's real exposure β is above the threshold $\frac{\sigma_\epsilon}{\sigma_z}$ and the lower bound of the disclosed interval β_L is also set above the threshold, then the ambiguity averse investors will choose the lower bound β_L as their risk perception. Because the firm prefers higher risk perception in this case, so it is optimal for it to set the highest possible lower bound at the true value, i.e., $\beta_L = \beta$. The upper bound in this case has no effect on investors' choice, so the firm can set any upper bound not below β. Similar argument can be applied to the case if the firm's real exposure $\beta \leq \frac{\sigma_\epsilon}{\sigma_z}$. Assuming that firms would be as precise as possible if they are indifferent, then this implies that it is optimal for firms to provide perfect risk disclosure for any value of true exposure, i.e., $\beta_L^* = \beta_H^* = \beta$ for $\forall\beta$.

Finally, given the optimal risk disclosure strategy at $t = 2$, the firm chooses the amount of private information about u, i.e., σ_u^2 to maximize its expected utility (averaging over e). Based on the expression of Expression (3.6), the firm's objective function at $t = 1$ is:

$$\begin{aligned} \max_{\sigma_u^2} E(U_f|\beta) &= E(p|\beta) - \frac{\gamma_f Var(p|\beta)}{2} \\ &= \beta\bar{z} - \frac{\gamma_f\left(\beta^2\sigma_z^2 + \sigma_u^2\right)}{2} - constant \end{aligned} \tag{B.4}$$

with the constant being $\frac{\gamma_f\hat{\beta}}{\theta_z+\hat{\beta}^2\theta_\epsilon} + \left(\frac{\gamma_f\hat{\beta}}{\theta_z+\hat{\beta}^2\theta_\epsilon}\right)^2\sigma_q^2$. It is optimal for the firm to retain *no* private information about u, i.e., $\sigma_u^2 = 0$.

To summarize, when the firm cares about the stock price, it is still optimal to give perfect disclosure of β and u, the same as in the benchmark case when the firm only cares about the terminal value.

Appendix C

Bibliography

Amador, M. and Weill, P.-O. (2010). Learning from prices: Public communication and welfare. *Journal of Political Economy*, 118(5):866–907.

Bai, J., Philippon, T., and Savov, A. (2016). Have financial markets become more informative? *Journal of Financial Economics*, 122(3):625–654.

Bakke, T.-E. and Whited, T. M. (2010). Which firms follow the market? an analysis of corporate investment decisions. *The Review of Financial Studies*, 23(5):1941–1980.

Bao, Y. and Datta, A. (2014). Simultaneously discovering and quantifying risk types from textual risk disclosures. *Management Science*, 60(6):1371–1391.

Bond, P., Edmans, A., and Goldstein, I. (2012). The real effects of financial markets. *Annu. Rev. Financ. Econ.*, 4(1):339–360.

Boot, A. W. and Thakor, A. V. (2001). The many faces of information disclosure. *The Review of Financial Studies*, 14(4):1021–1057.

Chen, Q., Goldstein, I., and Jiang, W. (2006). Price informativeness and investment sensitivity to stock price. *The Review of Financial Studies*, 20(3):619–650.

Chen, Q., Huang, Z., and Zhang, Y. (2014). The effects of public information with asymmetrically informed short-horizon investors. *Journal of Accounting Research*, 52(3):635–669.

Demski, J. S. and Feltham, G. A. (1994). Market response to financial reports. *Journal of Accounting and Economics*, 17(1-2):3–40.

Diamond, D. W. (1985). Optimal release of information by firms. *The journal of finance*, 40(4):1071–1094.

Dye, R. A. and Sridhar, S. (2002). Resource allocation effects of price reactions to disclosures. *Contemporary Accounting Research*, 19(3):385–410.

Dyer, T., Lang, M., and Stice-Lawrence, L. (2017). The evolution of 10-k textual disclosure: Evidence from latent dirichlet allocation. *Journal of Accounting and Economics*, 64(2-3):221–245.

Edmans, A., Jayaraman, S., and Schneemeier, J. (2017). The source of information in prices and investment-price sensitivity. *Journal of Financial Economics*, 126(1):74–96.

Fischer, P. E. and Stocken, P. C. (2010). Analyst information acquisition and communication. *The Accounting Review*, 85(6):1985–2009.

GAO (2018). Climate-related risks: Sec has taken steps to clarify disclosure requirements. `https://www.gao.gov/products/GAO-18-188`.

Gao, P. and Liang, P. J. (2013). Informational feedback, adverse selection, and optimal disclosure policy. *Journal of Accounting Research*, 51(5):1133–1158.

Goldstein, I. and Yang, L. (2015). Information diversity and complementarities in trading and information acquisition. *The Journal of Finance*, 70(4):1723–1765.

Han, B. and Yang, L. (2013). Social networks, information acquisition, and asset prices. *Management Science*, 59(6):1444–1457.

Hayek, F. A. (1945). The use of knowledge in society. *The American economic review*, 35(4):519–530.

Heinle, M. S. and Smith, K. C. (2017). A theory of risk disclosure. *Review of Accounting Studies*, 22(4):1459–1491.

Heinle, M. S., Smith, K. C., and Verrecchia, R. E. (2018). Risk-factor disclosure and asset prices. *The Accounting Review*, 93(2):191–208.

Hellwig, C. and Veldkamp, L. (2009). Knowing what others know: Coordination motives in information acquisition. *The Review of Economic Studies*, 76(1):223–251.

Indjejikian, R. J. (1991). The impact of costly information interpretation on firm disclosure decisions. *Journal of Accounting Research*, 29(2):277–301.

Jayaraman, S. and Wu, J. S. (2018). Is silence golden? real effects of mandatory disclosure. *The Review of Financial Studies*, 32(6):2225–2259.

Jorgensen, B. N. and Kirschenheiter, M. T. (2003). Discretionary risk disclosures. *The Accounting Review*, 78(2):449–469.

Kim, O. and Verrecchia, R. E. (1994a). Market liquidity and volume around earnings announcements. *Journal of accounting and economics*, 17(1-2):41–67.

Kim, O. and Verrecchia, R. E. (1994b). Market liquidity and volume around earnings announcements. *Journal of accounting and economics*, 17(1-2):41–67.

Kravet, T. and Muslu, V. (2013). Textual risk disclosures and investors risk perceptions. *Review of Accounting Studies*, 18(4):1088–1122.

Lin, Y. (2019). Soft and ambiguous risk disclosure. Working paper.

Luo, Y. (2005). Do insiders learn from outsiders? evidence from mergers and acquisitions. *The Journal of Finance*, 60(4):1951–1982.

McNichols, M. and Trueman, B. (1994). Public disclosure, private information collection, and short-term trading. *Journal of Accounting and Economics*, 17(1-2):69–94.

Smith, K. (2019). Risk disclosure, liquidity, and investment efficiency. Working paper.

Xue, H. and Zheng, R. (2018). Word of mouth, noise-driven volatility, and public disclosure. Working Paper.

Yan, H. (2019). *The Real Effects of Algorithmic Trading*. PhD thesis, Duke University.

Zuo, L. (2016). The informational feedback effect of stock prices on management forecasts. *Journal of Accounting and Economics*, 61(2-3):391–413.